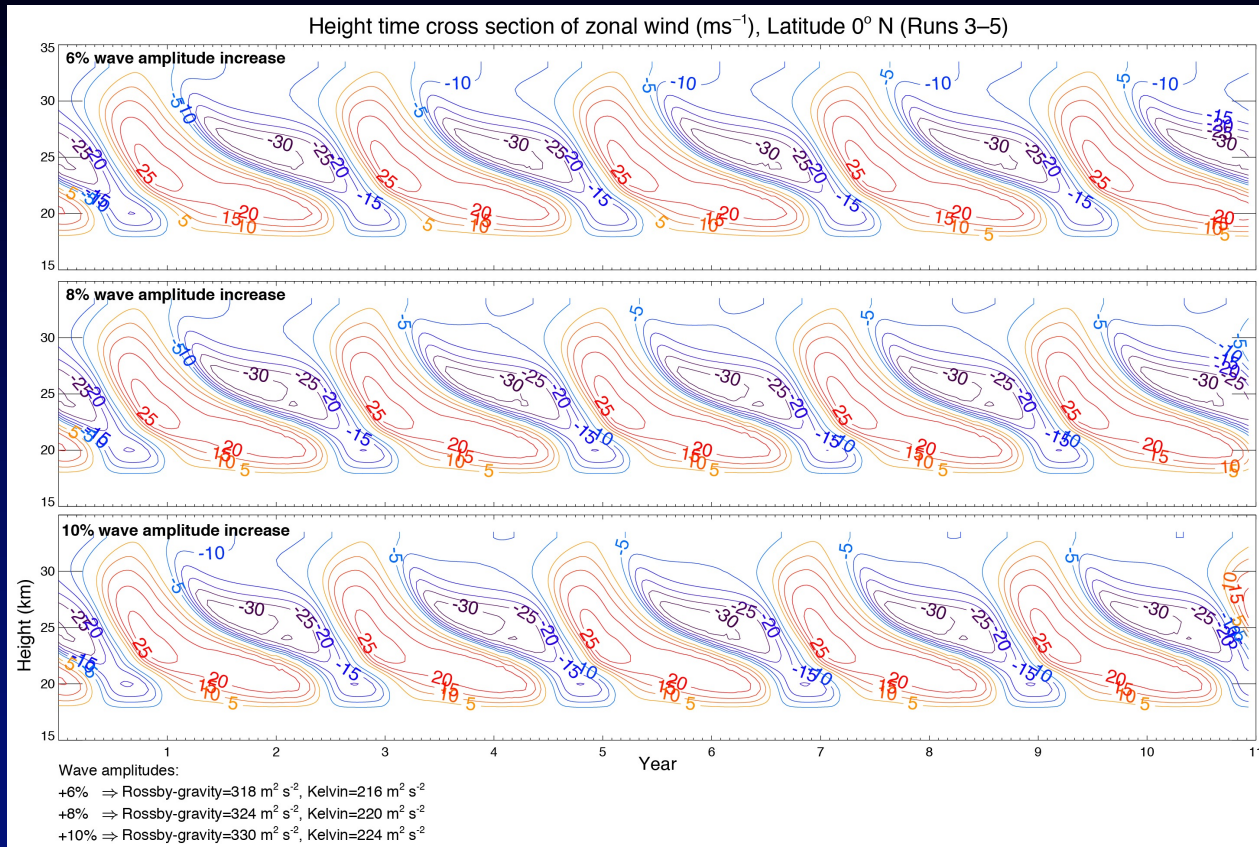


# Numerical Simulations of Tropospheric Heating Effects on the Quasi-biennial Oscillation



John Noble  
Meteorology 205A  
December 4, 2006  
(Revised, 2024)

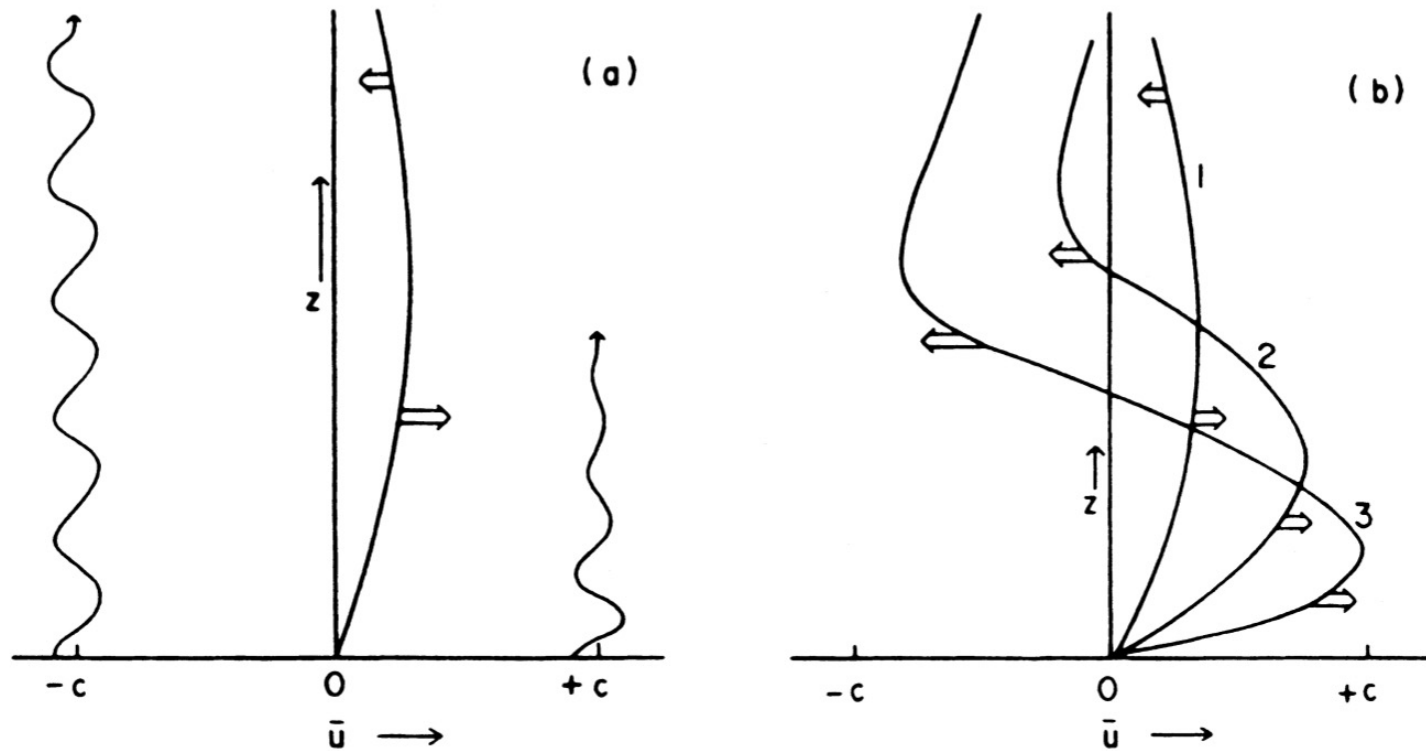
# Introduction

The quasi-biennial oscillation (QBO) is an oscillation of easterly and westerly stratospheric zonal wind regimes with a period range of 22–34 months and a mean period of 28 months (Baldwin *et al.* 2001; Holton 2004).

Both observations and theoretical analysis have identified that vertically propagating equatorial Kelvin and Rossby-gravity waves drive the westerly and easterly QBO regimes respectively (Holton 2004). The QBO is a unique dynamical phenomenon in that its period is seemingly unrelated to the periods of these driving waves (Baldwin 2001).

This presentation describes results from 2.5-dimension model simulations of the effects of tropospheric warming on the QBO. Warming was simulated by increasing the amplitudes of Kelvin and Rossby-gravity waves.

# Schematic representation of wave-driven accelerations that lead to the zonal wind QBO



**Fig. 12.16** Schematic representation of wave-driven accelerations that lead to the zonal wind QBO. Eastward and westward propagating gravity waves of phase speeds  $+c$  and  $-c$ , respectively, propagate upward and are dissipated at rates dependent on the Doppler-shifted frequency. (a) Initial weak westerly current selectively damps the eastward propagating wave and leads to westerly acceleration at lower levels and easterly acceleration at higher levels. (b) Descending westerly shear zones block penetration of eastward propagating waves, whereas westward propagating waves produce descending easterlies aloft. Broad arrows show locations and direction of maxima in mean wind acceleration. Wavy lines indicate relative penetration of waves. (After Plumb, 1982.)

**Table 12.1** *Characteristics of the Dominant Observed Planetary-Scale Waves in the Equatorial Lower Stratosphere*<sup>a</sup>

Theoretical description	Kelvin wave	Rossby-gravity wave
Discovered by	Wallace and Kousky (1968)	Yanai and Maruyama (1966)
Period (ground-based) $2\pi\omega^{-1}$	15 days	4–5 days
Zonal wave number $s = ka \cos \phi$	1–2	4
Vertical wavelength $2\pi m^{-1}$	6–10 km	4–8 km
Average phase speed relative to ground	+25 m s <sup>-1</sup>	-23 m s <sup>-1</sup>
Observed when mean zonal flow is	Easterly (maximum $\approx -25$ m s <sup>-1</sup> )	Westerly (maximum $\approx +7$ m s <sup>-1</sup> )
Average phase speed relative to maximum zonal flow	+50 m s <sup>-1</sup>	-30 m s <sup>-1</sup>
Approximate observed amplitudes		
$u'$	8 m s <sup>-1</sup>	2–3 m s <sup>-1</sup>
$v'$	0	2–3 m s <sup>-1</sup>
$T'$	2–3 K	1 K
Approximate inferred amplitudes		
$\Phi'/g$	30 m	4 m
$w'$	$1.5 \times 10^{-3}$ m s <sup>-1</sup>	$1.5 \times 10^{-3}$ m s <sup>-1</sup>
Approximate meridional scales $(2N/\beta m )^{1/2}$	1300–1700 km	1000–1500 km

<sup>a</sup>(After Andrews et al., 1987).

# Model description

- Dynamics based on Takahashi's 2-D model (1987)
- Cordero and Nathan (2000) extended the model by coupling the wave and zonal mean ozone continuity equations to the wave and zonal mean temperature equations respectively through diabatic heating terms.
  - ⇒ Enables the model to simulate the effects of the wave fields on the zonal mean circulation
- Zonal mean and linear perturbation expressions of the primitive equations are used to model the atmosphere
- Circulation is driven by the geopotential height of Kelvin and Rossby waves (KW and RW) at the lower boundary

# Zonal mean equations

The zonal mean equations include:

$$\frac{\partial \bar{u}}{\partial t} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{w} \frac{\partial \bar{u}}{\partial z} - \beta y \bar{v} = -\frac{\partial}{\partial y} (\overline{u'v'}) - \frac{1}{\rho} \frac{\partial}{\partial z} (\overline{\rho u'w'}) + \nabla_D^2 \bar{u}, \quad (1)$$

$$\frac{\partial \bar{v}}{\partial t} + \beta y \bar{u} = -\frac{\partial \bar{\Phi}}{\partial y} + \nabla_D^2 \bar{v}, \quad (2)$$

$$\frac{\partial \bar{v}}{\partial y} + \frac{1}{\rho} \frac{\partial (\rho \bar{w})}{\partial z} = 0, \quad (3)$$

$$\frac{\partial \bar{\Phi}_z}{\partial t} + \bar{v} \frac{\partial \bar{\Phi}_z}{\partial y} + \bar{w} \frac{\partial \bar{\Phi}_z}{\partial z} + N^2 \bar{w} = -\frac{\partial (\overline{v' \Phi'_z})}{\partial y} - \alpha_N (\bar{\Phi}_z) + A \bar{\gamma} + \nabla_D^2 \bar{\Phi}_z. \quad (4)$$

Geopotential is related to temperature by

$$T = \frac{H}{R} \frac{\partial \Phi}{\partial z}. \quad (5)$$

# Zonal mean equations

The above equations are zonal mean prognostic equations:

- (1)  $u$ -momentum,
- (2)  $v$ -momentum,
- (3) mass continuity,
- (4) geopotential, which can be used as a prognostic equation for temperature due to (5).

The first term on the right-hand side (RHS) of (4) is northward heat flux.

# Boundary conditions

Circulation in the model is driven by the geopotential height of equatorial Kelvin and Rossby-gravity waves at the lower boundary represented by

$$\Phi'_K = A_K \exp\left(-\frac{\beta y^2}{2c_K}\right) \text{Re}\left[e^{ik_K(x-c_K t)}\right], \quad (6)$$

where subscript  $K$  refers to Kelvin waves,

$c_K$  = phase speed,

$k_K$  = zonal wavenumber,

$m_K$  = vertical wavenumber,

$A_K$  = specified wave amplitude, and

$$\Phi'_R = A_R y \left(\frac{\beta m_R}{N}\right)^{1/2} e^{-\frac{\beta |m_R| y^2}{2N}} \times \text{Re}\left[e^{ik_R(x-c_R t)}\right], \quad (7)$$

where subscript  $R$  refers to Rossby-gravity waves.



# Tropospheric warming

- Tropospheric temperatures can not be specified
- Increasing the amplitudes of the Kelvin and Rossby-gravity waves is used to simulate tropospheric warming.

# QBO characteristics

- Irregular period
  - ❖ ~28 month mean
  - ❖ 22–34 month range
- Easterly velocity  $>$  westerly
- Westerly regime duration  $>$  easterly  
(Andrews *et al.* 1987)

Time–height section of departure of monthly mean zonal winds ( $\text{m s}^{-1}$ ) for each month from the long-term average

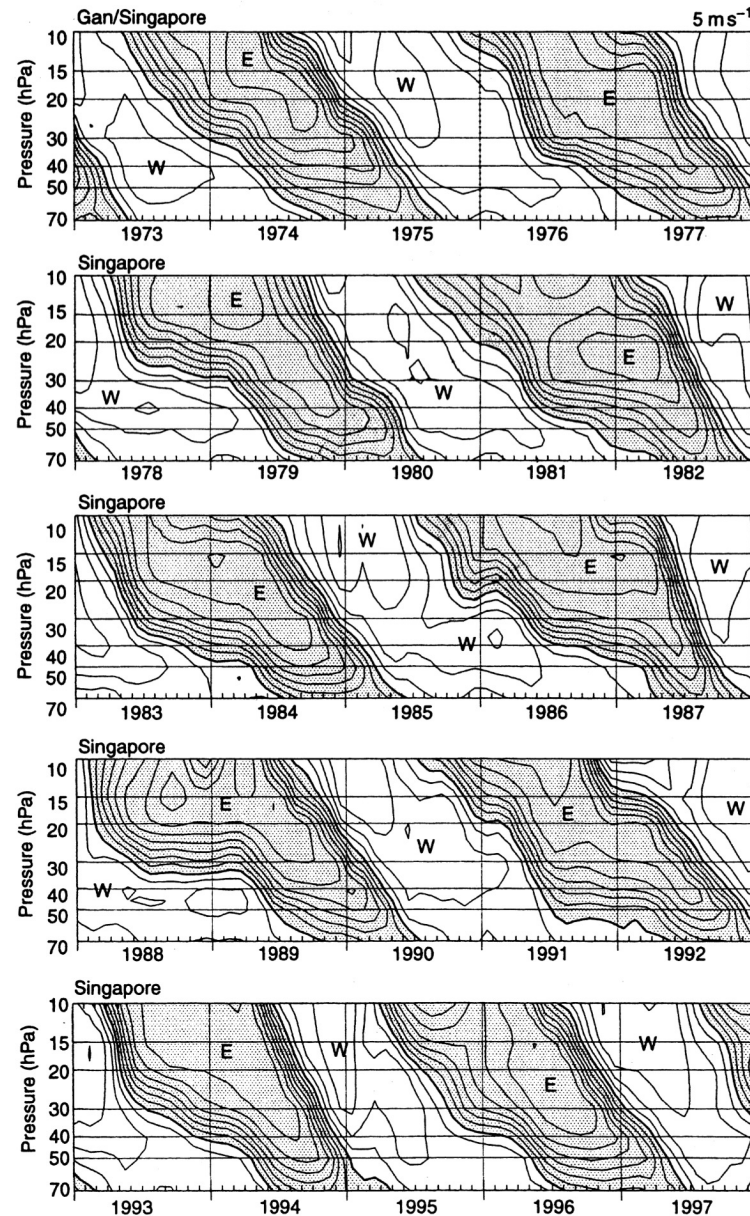


Fig. 12.15 Time–height section of departure of monthly mean zonal winds ( $\text{m s}^{-1}$ ) for each month from the long-term average for that month at equatorial stations. Note the alternating downward propagating westerly (W) and easterly (E) regimes. (After Dunkerton, 2003. Data provided by B. Naujokat.)

# Mechanisms

Momentum deposition associated with damped vertically propagating equatorial waves drives the mean flow

- Kelvin wave: westerly momentum
- Rossby-gravity wave: easterly momentum

(Holton and Lindzen 1972)

# Equatorial stratospheric waves

- Vertically propagating equatorial waves share a number of physical properties with ordinary gravity waves (Holton 2004)
- Inertia-gravity waves can propagate vertically only when the wave frequency satisfies the inequality:

$$f < \nu < N$$

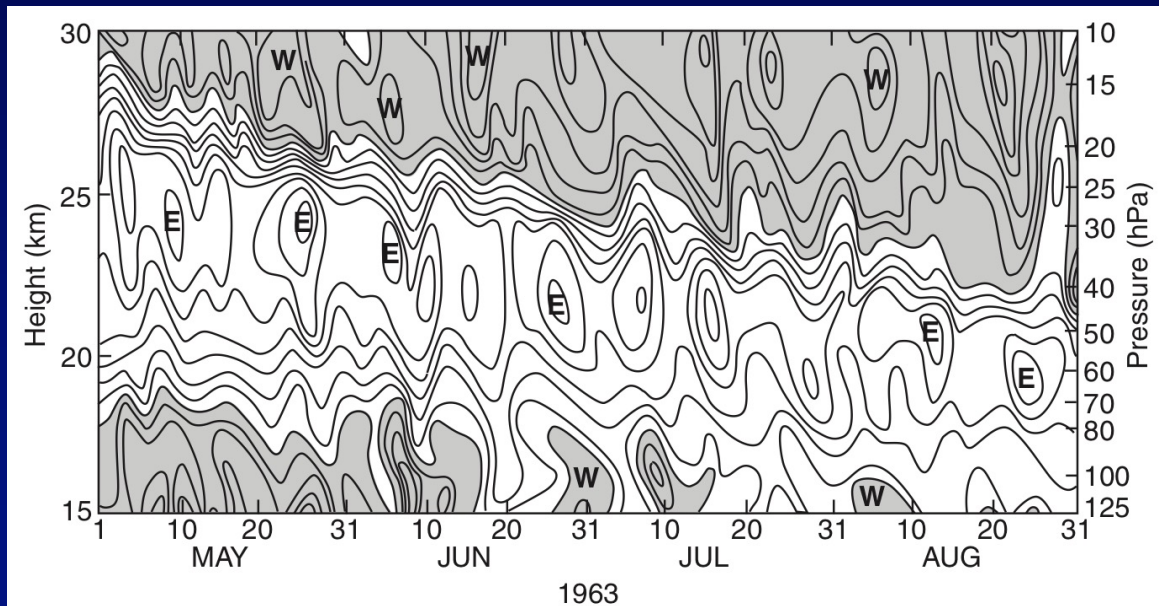
# Kelvin waves

- At the equator, eastward propagating waves with negligible meridional velocity component and Gaussian latitudinal structure in zonal velocity, geopotential, and temperature, symmetric about the equator (Baldwin 2001)
- Amplitude is significant only within  $20^\circ$  of the equator (Holton 2004)

# Vertically propagating Kelvin waves

Characteristics (Holton 2004):

- Primarily of zonal wave number  $s = 1$
- Periods in the range of 12–20 days
- Temperature oscillation leads zonal wind by 1/4 cycle
  - This is the phase relationship required for upward propagating Kelvin waves



**Fig. 12.14** Time–height section of zonal wind at Canton Island ( $3^{\circ}\text{S}$ ). Isotachs at intervals of  $5 \text{ m s}^{-1}$ . Westerlies are shaded. (Courtesy of J. M. Wallace and V. E. Kousky.)

(Holton 2004)

# Rossby-gravity waves

- Existence of Rossby-gravity waves has been confirmed in observations (Holton 2004)
- Characteristics:  $s = ka \cos \phi$ 
  - Zonal wave number:  $s = 4$ , where
  - Vertical wavelength in the range of 6–12 km
  - Period range of 4–5 days



# Wave formation

Kelvin and Rossby-gravity waves are excited by oscillations in large scale convective heating in the equatorial troposphere

# Transformed Eulerian mean (TEM) equation

- The combined effect of eddy forcing on transport in the meridional plane takes into account the strong cancellation between the eddy fluxes and the meridional terms through the concept of the residual mean meridional circulation.
- TEM equations were originally formulated by Andrews and McIntyre (1976)
- Kelvin & Rossby Gravity waves affect the mean flow
- Wave forcing term: forcing from K & RG waves

$$\frac{\partial \bar{u}}{\partial t} = f_0 \bar{v}^* + \frac{1}{\rho_0} \nabla \cdot \vec{F} + X \quad (8)$$

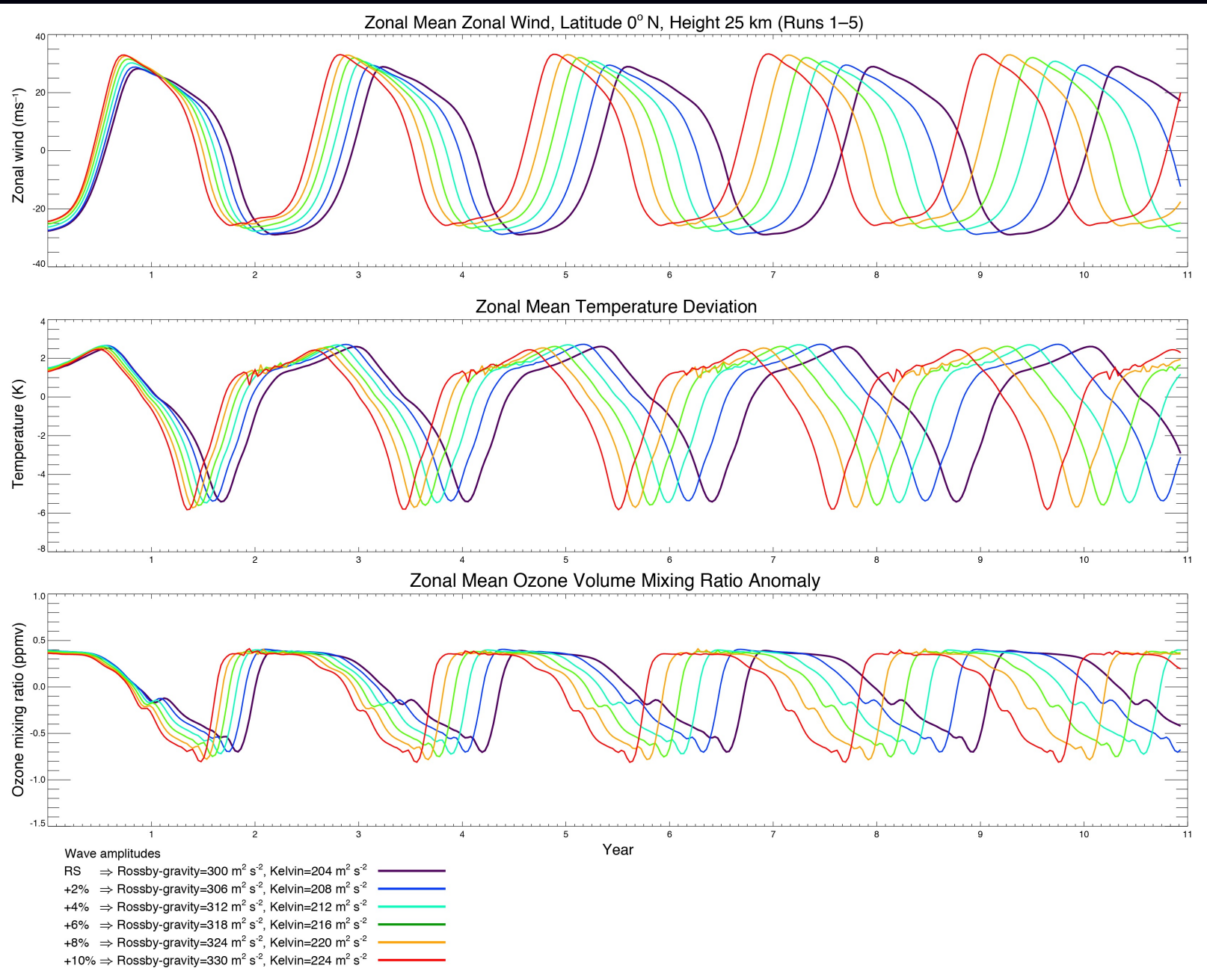
# Methods

The effects of tropospheric warming on the QBO were simulated by varying the Kelvin and Rossby-gravity wave amplitudes ( $A_K$ ,  $A_R$ ). As a first order simulation, five model runs were conducted in which the amplitudes of both the Kelvin and Rossby-gravity waves were increased successively by 2% from the RS values (Table 1).

Table 1. Run parameters

Run number	Percent increase from RS	Rossby-gravity wave Amplitude ( $\text{m}^2 \text{s}^{-2}$ )	Kelvin wave Amplitude ( $\text{m}^2 \text{s}^{-2}$ )
RS	0	300	204
1	2	306	208
2	4	312	212
3	6	318	216
4	8	324	220
5	10	330	224

# Figure 1. Zonal mean wind, temperature, and ozone (Runs 1–5)



# Results, Figure 1

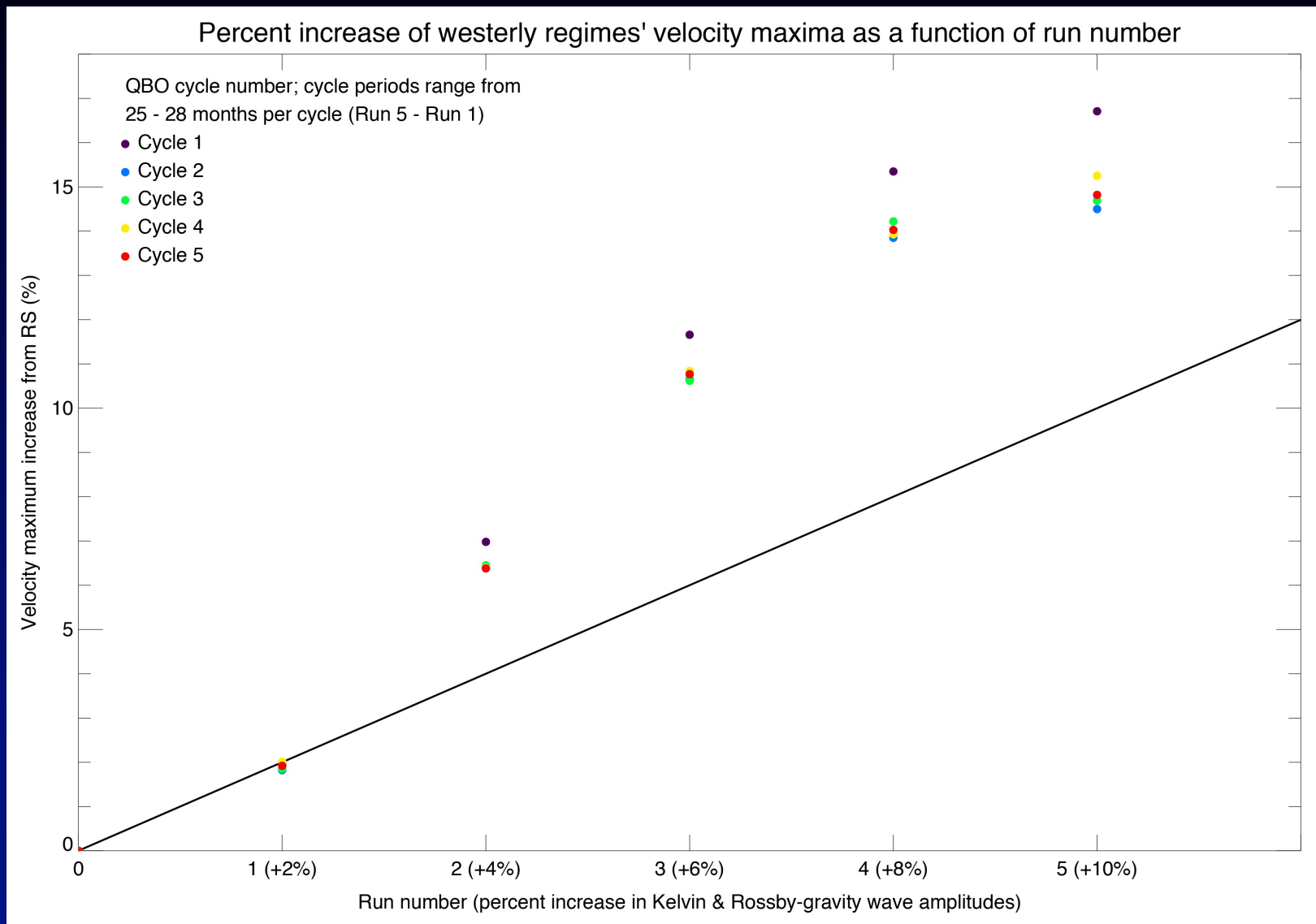
Zonal wind, temperature, and ozone QBO plots were generated to compare the different runs.

Figure 1 shows overlays of the five runs for the above three quantities. The zonal wind QBO indicates that the westerly regime's maximum velocities increased with increasing run number, from 29.35 to 33.18 m s<sup>-1</sup> for runs 1 to 5 respectively while the easterly regime's maximum velocities decreased in magnitude from -28.84 to -25.74 for runs 1 to 5 respectively.

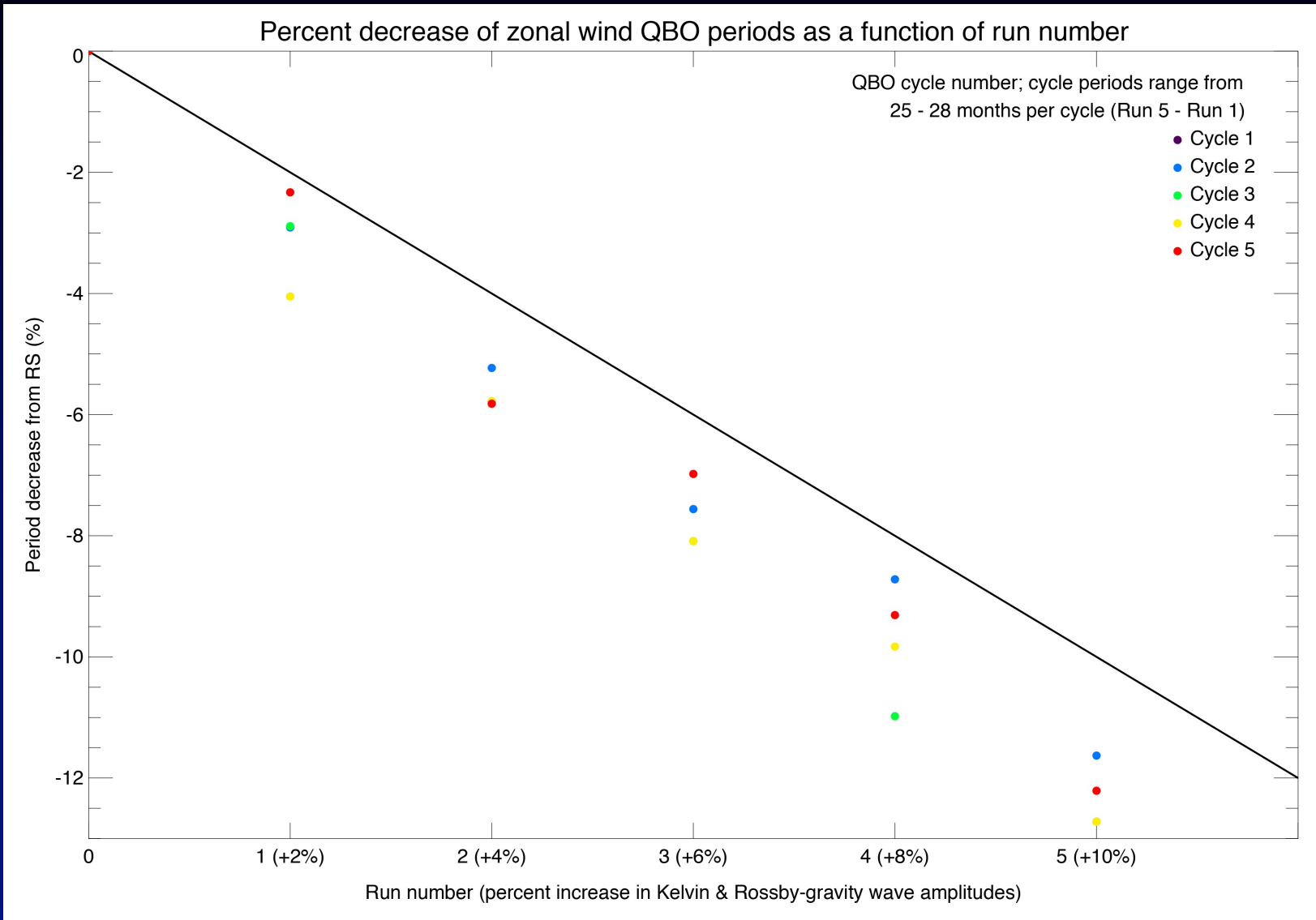
The mean *u*-wind QBO amplitude, however, increased only slightly, from 29.07 to 29.46 m s<sup>-1</sup> for runs 1 to 5 respectively, reflecting the symmetric trend of increasing westerly and decreasing easterly magnitudes for all runs.

Period changes in the temperature and ozone QBOs were similar to those seen in the *u*-wind QBOs (Figure 1). The above changes can also be seen in Figure 4 and 5: time-height cross sections of zonal wind for runs 1–5 (westerly winds are represented in red, easterly in blue).

# Figure 2. Percent increase of westerly regimes' velocity maxima



# Figure 3. Percent decrease of zonal wind QBO periods



# Results, Figures 2 and 3

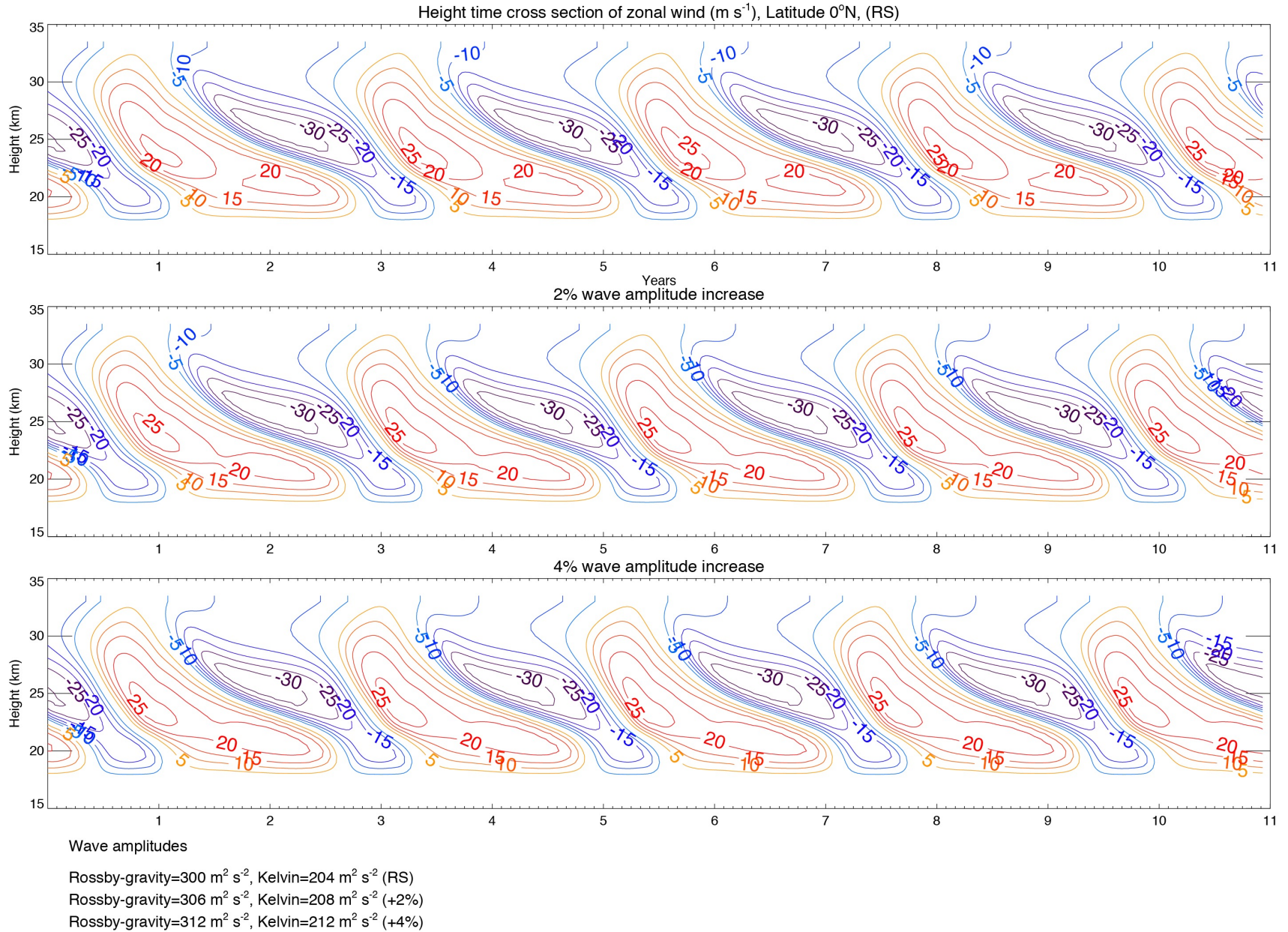
Figure 2 shows the percent increase of the westerly regimes' velocity maxima as a function of run number. The solid line represents a linear increase from 0 to 10%, while the dots represent the maximum westerly velocity percent increase from the RS for the five QBO cycles in the 11-year simulation. While run 1 displays a near linear velocity increase for the five cycles, with percent increases from 1.94 to 1.92, runs 2–5 display non-linear velocity maxima increases for each of the five QBO cycles, with the greatest deviation (from linear) occurring in the first QBO cycle of run 5 (16.7 % increase).

Figure 3 shows the percent decrease of zonal wind QBO periods as a function of run number, indicating a non-linear change in almost all cases.

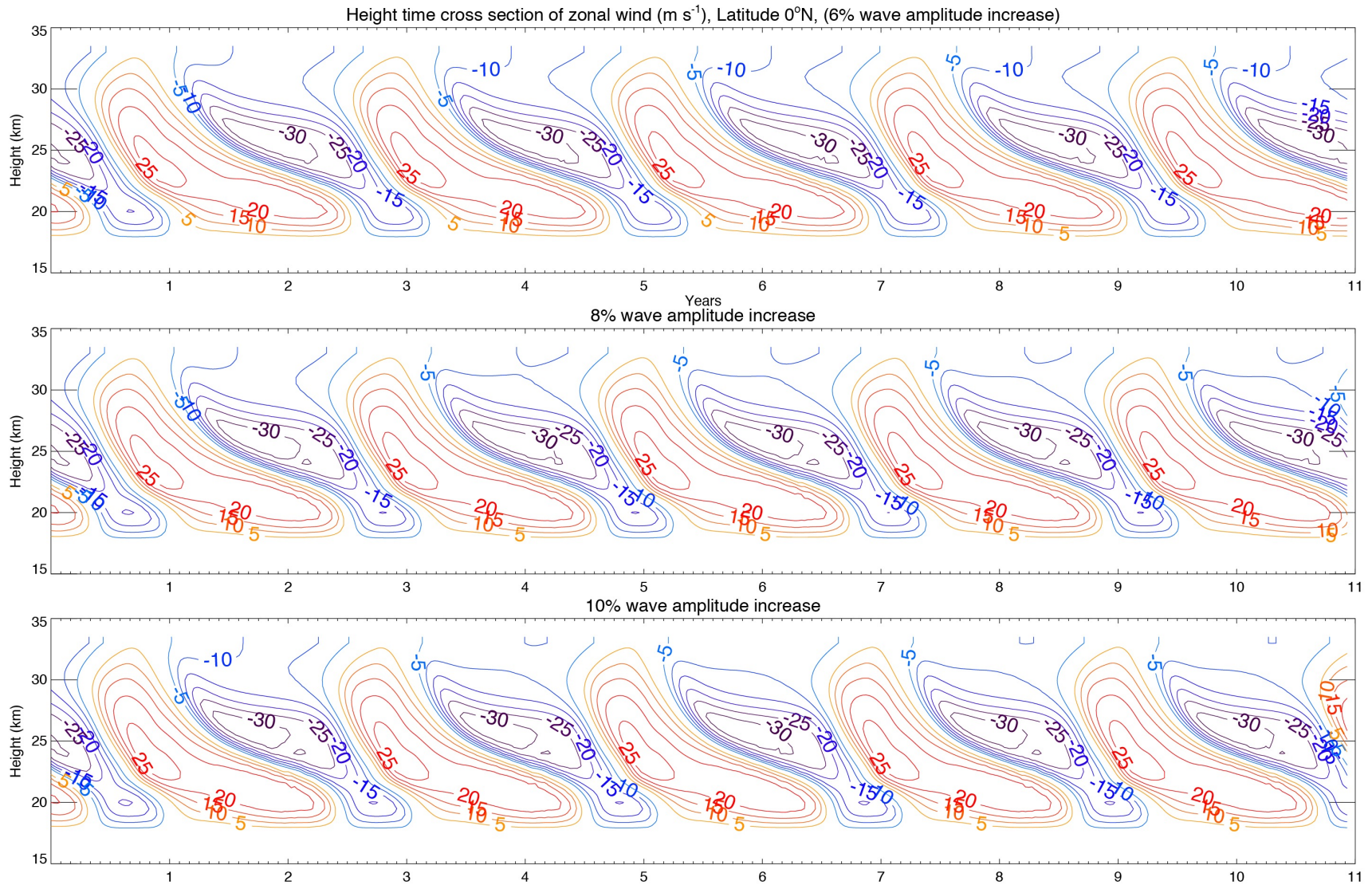
These non-linear changes appear to be a result of the non-linear, coupled, equations which govern the mean flow. Changes in the wave amplitude via equations (6) and (7) affect (4) which in turn affects the zonal flow in (1) via  $\bar{v}$  and  $\bar{w}$ .



# Figure 4. Height time cross section of zonal wind (Runs 1 & 2)



# Figure 5. Height time cross section of zonal wind (Runs 3–5)



Wave amplitudes

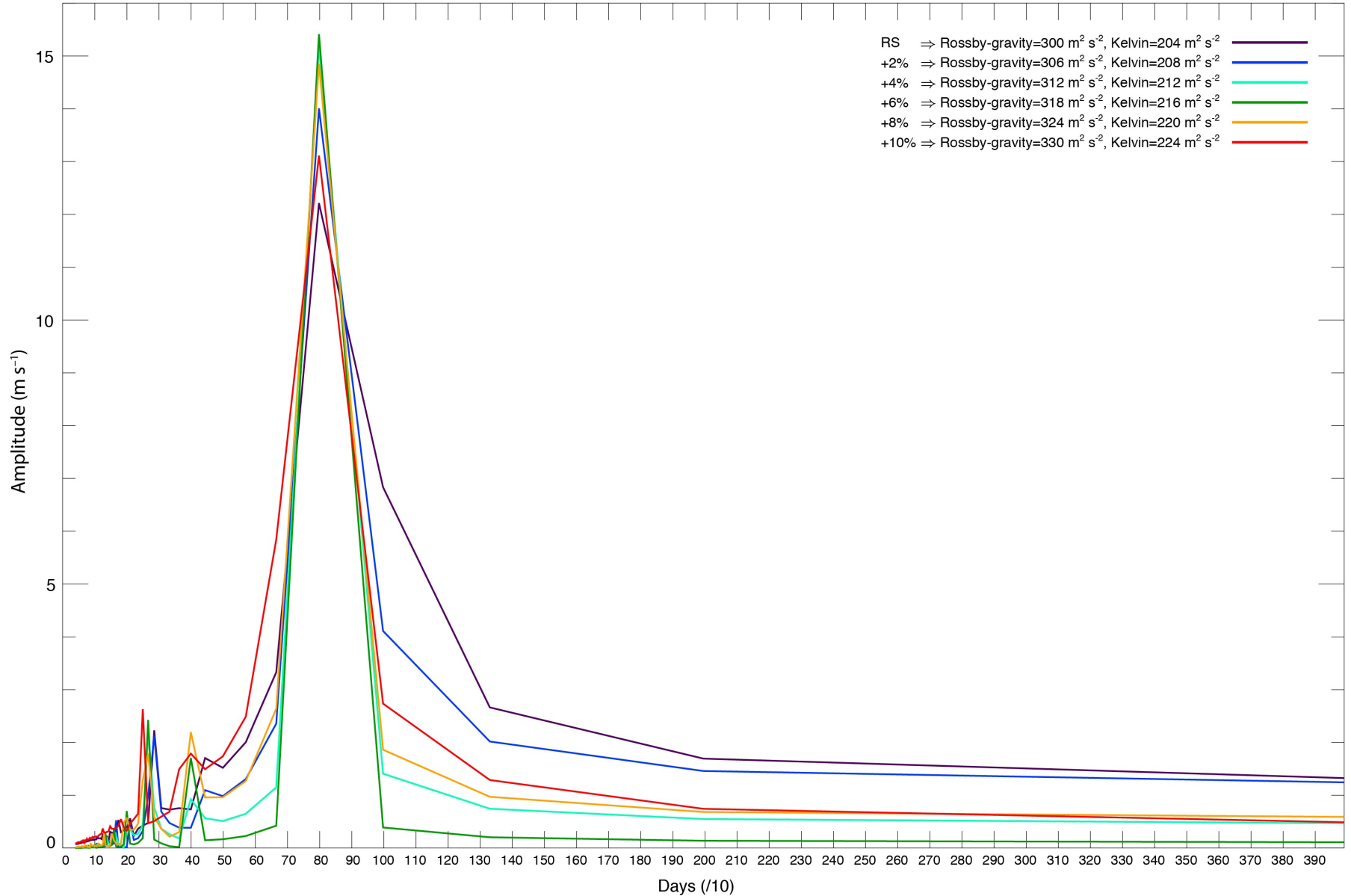
Rossby-gravity= $318 \text{ m}^2 \text{ s}^{-2}$ , Kelvin= $216 \text{ m}^2 \text{ s}^{-2}$  (+6%)

Rossby-gravity= $324 \text{ m}^2 \text{ s}^{-2}$ , Kelvin= $220 \text{ m}^2 \text{ s}^{-2}$  (+8%)

Rossby-gravity= $330 \text{ m}^2 \text{ s}^{-2}$ , Kelvin= $224 \text{ m}^2 \text{ s}^{-2}$  (+10%)

Figure 6. Fast Fourier transform power spectra of zonal wind, Latitude 0° N

Fast Fourier transform power spectra of zonal wind, Latitude 0° N (Runs 1–5)



# Discussion

Both observations and theoretical analysis have determined that vertically propagating Kelvin and Rossby-gravity waves provide a significant portion of the momentum required to drive the QBO with Kelvin waves transferring westerly momentum and Rossby-gravity waves transferring easterly momentum (Holton 2004).

The consistent increase in westerly winds and decrease in easterly winds is an interesting result. This may be a function of how the model is tuned, resulting in Kelvin waves dominating Rossby-gravity waves (Cordero and Nathan 2000).

Further simulations which isolate the effects of both Kelvin and Rossby-gravity waves are required to better understand these dynamics.

# References

- Andrews, D. G., J. R. Holton, and C. B. Leovy, 1987: *Middle Atmosphere Dynamics*. Academic Press Inc., Orlando, FL.
- Baldwin, M. P., L. J. Gray, T. J. Dunkerton, K. Hamilton, P. H. Haynes, W. J. Randel, J. R. Holton, M. J. Alexander, I. Hirota, T. Horinouchi, D. B. A. Jones, J. S. Kinnersley, C. Marquardt, K. Sato, and M. Takahashi, 2001: The quasi-biennial oscillation. *Rev. Geophys.*, **39**, 179–229.
- Cordero, E. C., and T. R. Nathan, 2000: The Influence of Wave- and Zonal Mean-Ozone Feedbacks on the Quasi-biennial Oscillation. *J. Atmos. Sci.*, **57**, 3426–3442.
- Holton, J.R. and Lindzen, R.S., 1972: An updated theory for the quasi-biennial cycle of the tropical stratosphere. *J. Atmos. Sci.*, **29**, 1076–1080.
- Holton, J. R., 2004: *An introduction to dynamic meteorology*. Elsevier Academic Press, 529 pp.
- Rind, D., D. Shindell, P. Lonergan, N. K. Balachandran, 1998: Climate change and the middle atmosphere. Part III: The doubled CO<sub>2</sub> climate revisited. *J. Climate*, **11**, 876–894.
- Takahashi, M., 1987: A 2-dimensional numerical model of the quasi-biennial oscillation: Part I. *J. Meteor. Soc. Japan*, **65**, 523–536.
- Takahashi, M., B. A. Boville, 1992: A three-dimensional simulation of the equatorial quasi-biennial oscillation. *J. Atmos. Sci.*, **49**, 1020–1035.