RADIATION : $\lambda = \dfrac{c}{\nu}$ | ν = frequency $\left[\mathrm{s^{-1}}\right]$ | **Wave Number** $\dfrac{\nu}{c} = \dfrac{1}{\lambda}$

Plank's Law : $E_\lambda^* = \dfrac{c_1}{\lambda^5\left[\exp\left(c_2/\lambda T\right)-1\right]}$ | $c_1 = 3.74\times10^{-16}$ W m^2, $c_2 = 1.44\times10^{-2}$ m °K

Radiance (L) **& Irradiance** (E)**:** $L = \dfrac{dE}{\cos\theta\, d\varpi}$, $E = \int_0^{2\pi} L\cos\theta\, d\varpi$ | θ= zenith ∠

Isotropic Radiation : $\boxed{E = \pi L}$

Total Irradiance : $E = \int_\lambda E_\lambda d\lambda \Rightarrow \boxed{E = \int_0^\infty E_\lambda d\lambda}$

Irradiance of Planetary rad emitted to space : $E = (1-A)\cdot \overline{E}_s/4$

Stephan Boltzman Law : $E^* \equiv \sigma T^4$ $\left[\mathrm{W\cdot m^{-2}}\right]$ | • Radiance emitted (by a black body (BB)) per unit area

Wien's Displacement Law : $\lambda_m = \dfrac{\alpha}{T}$ | α=2898 μm K; λ of peak transmission for BB at T

Inverse Square Law : $\dfrac{\overline{E}_s}{E_0} = \left(\dfrac{R_{sun}}{R_{Earth\, orbit}}\right)^2$ $\left[\mathrm{W\cdot m^{-2}}\right]$

Kirchhoff's Law : $a_\lambda = \varepsilon_\lambda$

Absorbtivity, Reflectivity, Transmissivity :

$a_\lambda + r_\lambda + \tau_\lambda = 1$ | For a black body, $r_\lambda = \tau_\lambda = 0$, and $a_\lambda = 1$ for all λ (spectral/monochromatic absorptance, reflectance, transmittance)

Clouds: SW: high reflectivity, low absorptivity. LW: high abs & high emis $a_\lambda = \varepsilon_\lambda$

Single scattering Albedo : $\varpi = k_s/k$

Beer's Law :

$$da_\lambda \equiv -\frac{dE_\lambda}{E_\lambda} = -k_\lambda \rho \sec\theta dz \;\Rightarrow\; \ln E_{\lambda\infty} - \ln E_\lambda = \sec\theta \int_z^\infty k_\lambda \rho\, dz \;\Rightarrow$$

$\boxed{E_\lambda = E_{\lambda\infty} e^{-\sigma_\lambda}}$ | where $\sigma_\lambda = \sec\theta \int_z^\infty k_\lambda \rho\, dz$

$u = \int_0^x \rho dx$ | mass of absorbing gas in a unit area

Optical Thickness/Depth : $k_\lambda u \equiv \int_0^x k_\lambda \rho_c dx$ [unitless] | k_λ = absorption coefficient of layer $\left[\mathrm{m^2\ kg^{-1}}\right]$; ρ_c = density of absobing constituent

Aerosols: 0.01 - 1, mean: 0.1; Clouds:10+

Monochromatic/Spectral Transmissivity : $\tau_\lambda \equiv \dfrac{L_\lambda}{L_{\lambda 0}} = e^{-k_\lambda u}$

Monochromatic/Spectral Absorptivity : $a_\lambda = 1 - e^{-k_\lambda u}$

| Can be used for radiance or irradiance, independent of direction

Radiative Equilibrium : Receive on a cross section $\left(\pi r^2\right)$, emit from the whole $\left(4\pi r^2\right)$

Rad In = Rad out $\Rightarrow \boxed{a_{sw}\cdot E_s \cdot \pi r^2 + a_{lw}\sigma_{SB} T_e^4 \pi r^2 = \sigma_{SB} T_B^4 4\pi r^2}$

$\boxed{T_p = \left[\dfrac{(1-A)E_p}{4\sigma_{SB}}\right]^{1/4}}$ | Where T_p = equilibrium temp of a planet and E_p = the solar constant for that planet

Spectral Curves : Although the irradiance absorbed by the earth & atm is closely equal to that emitted, and although both distributions are very roughly black in character, the spectral curves of absorbed & emitted radiation overlap almost not at all.

Satellites : $\varpi = 2\pi\left[1 - \dfrac{\left(2Rh + h^2\right)^{1/2}}{R+h}\right]$ | Shows that ϖ is a function of height only. Derived from Pythagorean & Solid ∠ eqns.

a_λ = Monochromatic Absorptivity $= 1 - \tau_\lambda = 1 - e^{-\sigma_\lambda} = 1 - e^{-k_\lambda u}$

c = speed of light = 2.998×10^8 m s^{-1}

f = Radiant flux [W] or $\left[\mathrm{J\ s^{-1}}\right]$

$E = F$ = Irradiance $\left[\mathrm{W\ m^{-2}}\right]$ or $\left[\mathrm{cal\ m^{-2}\ min^{-1}}\right]$

E_λ = Monochromatic Irradiance/Emittance $\left[\mathrm{W\ m^{-2}}/\mu\mathrm{m}\right]$

$\overline{E}_s = S$ = Solar constant = 1370 W m^{-2}

$L = I$ = Radiance (AKA Intensity) $\left[\mathrm{W\ m^{-2}\ sr^{-1}}\right]$

$k_\lambda = k_a = k_\lambda\left(T, p, \text{composition}\right)$ = Absorption coefficient $\left[\mathrm{m^2\ kg^{-1}}\right]$

k_s = scattering coefficient; $k_s = N\pi r^2 Q_{sca.}$ $\left[\mathrm{m^2}\right]$ per 1 molecule:;

n = refractive index of particle

u = density weighted path length; mass in unit column = $\sec\theta \int \rho\, dz$ $\left[\mathrm{kg\ m^{-2}}\right]$

Q_{sca} = Scattering efiiciency factor [unitless]

T_e = Effective Radiation Emission Temperature

x = size parameter $2\pi r/\lambda$

ε = Emissivity $\equiv E_\lambda/E_\lambda^*$; GB: $E/E^* = E/\sigma T^4$; $0 < \varepsilon < 1$

θ = Zenith angle

ϕ = Latitude

$\sigma_\lambda = \sec\theta k_\lambda u$ = Optical depth/thickness [unitless]

$\sigma_\lambda = \sec\theta k_\lambda \int \rho\, dz$ = Optical depth/thickness [unitless]

$\sigma_{SB} = 5.67\times10^{-8}$ W m^{-2} K^{-4}

τ_λ = Monochromatic Transmissivity $\equiv E_\lambda/E_{\lambda\infty} = e^{-\sigma_\lambda} = e^{-k_\lambda u}$

ABSORBTION λ dependent

N_2O: 4-5 μm & ~7μm; CH_4: ~3μm & ~7μm; O_2: 0 – 0.3 μm; O_3: 9 – 10μm;

H_2O: ~1-3μm, 5-7μm, 12-20μm; CO_2: ~2μm & 4μm; 13μm-20μm

Atm window: 8-11μm

SCATTERING Key: size of particle to wavelength of light

$\boxed{k_s = N\pi r^2 Q_{sca}}$ | $N\pi r^2$ = Total Geometric cross section; N = # of molecules

x		Q_{sca}
2 – 14	Aerosols	
100	Cloud	2

I) Rayleigh : $2r/\lambda = 0.1$ (Air molecules), Scatterers are small compared to λ. Typically $r < 0.05\mu$m. Rayleigh also applies to scattering of radar beams by raindrops, where λ is 3-10cm & r is a few mm. $k_s \approx 1/\lambda^4$

Strongly λ dependent; shorter λ scatter most, $\therefore$ blue light scatters more than red

Assume isotropic sphere; medium's refractive index =1

$$k_s = N\pi r^2 \frac{128\pi^4 r^4}{3\lambda^4}\left(\frac{n^2-1}{n^2+2}\right)^2 \quad \text{since: } Q_{sca} = \frac{8}{3}x^4\left(\frac{n^2-1}{n^2+2}\right)^2 \ \& \ \boxed{x = \frac{2\pi r}{\lambda}}$$

$$\boxed{k_s = \frac{8\pi^3}{3\lambda^4 N_0^2}\left(n_0^2-1\right)^2 \frac{6+3\rho_n}{6-7\rho_n}}$$

k_s (per molecule) $\left[\text{cm}^2\right]$

Scattering cross section $= k_s \cdot N\left[\text{cm}^{-1}\right]$

Optical depth: τ = Scattering cross section$\left[\text{cm}^{-1}\right]\cdot \Delta u[\text{cm}] = \tau[\text{unitless}]$

II) Mie $\Rightarrow 0.1 < 2r/\lambda < 100$ (Dust, haze, smoke); not λ dependent

k_s (per molecule) $= \pi r^2 Q_{sca}$ $\left[\text{cm}^2\right]$

Scattering Cross Section $= N\pi r^2 Q_{sca}$ $\left[\text{cm}^{-1}\right]$

$$Q_{sca}(x \to \infty) = 1 + \left|\frac{m-1}{m+1}\right| \qquad \boxed{x = \frac{2\pi r}{\lambda}}$$

Optical depth: τ = Scattering cross section$\left[\text{cm}^{-1}\right]\cdot \Delta u[\text{cm}] = \tau[\text{unitless}]$

III) Geometric Optics $\Rightarrow 2r/\lambda > 100$ (Cloud droplets, ice)

NOTE: MAKE SURE YOU SQUARE THE RADIUS!

Optical depth is much greater for haze than for air molecules. Even though $N_{haze} \ll N_{air}$, Scat X section (haze) $\gg$ Scat X sect (air) & $\therefore$ $\tau_{haze} \gg \tau_{air}$

SOLID ANGLE : $\varpi \equiv \frac{A}{r^2}\left[\begin{matrix}\text{unitless, like rad}\\ \text{sr; steradian}\end{matrix}\right]$ | $\equiv$ ratio of the area of a sphere intercepted by the cone to the square of the sphere's radius.

$\therefore \varpi = 4\pi$ sr; $d\varpi_{\theta\phi} = \sin\theta\, d\phi\, d\theta$ | Used to determine radiation flux passing thru a unit area. In special case that the vert. axis is an axis of circular symmetry $\Rightarrow \boxed{d\varpi_\theta = 2\pi \sin\theta\, d\theta}$

$$\Rightarrow \varpi = 2\pi \int_0^\theta \sin\theta\, d\theta \Rightarrow \boxed{\varpi = 2\pi(1-\cos\theta)}$$

PHYSICAL CONSTANTS & QUANTITIES :

Sun : $T_{sun} = 5780$ K; $R_{sun} = 6.96\times10^8$ m; Sun's radiant flux: 3.9×10^{26} W

$3.9\times10^{26}\ \text{W}/4\pi r^2 = E_{sun} = E_0 = 6.328\times10^7\ \text{W m}^{-2}$; Can treat as || beam rad, don't $\int d\varpi$

Solar radiation range: λ: $(10^{-14}\text{m} \leftrightarrow 10^{10}\text{m})$; $\lambda_{\text{max emiss/intensity}}$: 0.475$\mu$m; v: $(10^{22}\ \text{s}^{-1} \leftrightarrow 10^{-2}\ \text{s}^{-1})$

Electromagnetic Rad : λ (m): AM radio: 100, TV: 1, Microwave: 10, IR: 10^{-6}, vis: 4-7.7$\times10^7$ (Violet: 0.4 μm, Red: 0.77 μm), UV: 10^{-7}, X-rays: 10^{-9}

$\uparrow$ T $\Rightarrow$ $\downarrow \lambda$ & $\uparrow$ in radiation emitted & $\uparrow$ in energy/wave or photon.

Earth : $R_{Earth} = 6.35677\times10^6$ m; $R_{\substack{earth\\orbit}} = 1.497\times10^{11}$ m; $T_{\substack{Earth's\\Eq.Temp}} = 255$ K (no atm.);

CONVERSION FACTORS

Temperature :

$\left(\frac{9}{5}\times°\text{C}\right)+32 = °\text{F}$; $(°\text{F}-32)\times\frac{5}{9} = °\text{C}$; K=°C +273.15

Area :

1 cm^2 = 10^{-4} m^2 $\Leftrightarrow$ 1 m^2 = 10^4 cm^2

Volume : V = 1 liter = 10^3cm^3 = 10^{-3}m^3

1 m^3 = 10^6cm^3 $\Leftrightarrow$ 1cm^3 = 10^{-6} m^3

Force :

1 Dyn = 10^{-5} N $\Leftrightarrow$ 1 N = 10^5 Dyn

Energy :

1 calorie = 4.18684×10^7 erg = 4.18684 Joule

1 Joule = 10^7 erg $\Leftrightarrow$ 1 erg = 10^{-7} J

1 J gm^{-1} = 1000 J kg^{-1}

Pressure :

1atm = 1013.25mb = 1013.25hPa = 101.325kPa = 1.01325×10^5Pa = 1.01325 bar

1 hPa = 100 Pa; 1 mb = 10^2 Pa

Density : $\rho = \frac{m}{V}$; $\alpha = \frac{V}{m}$; $\Rightarrow V = \alpha m$; $\therefore \rho = \frac{m}{\alpha m} = \frac{1}{\alpha}$; $\Rightarrow \alpha = \frac{1}{\rho}$

$\rho_w = 1\text{gm}\cdot\text{cm}^{-3}$; $\Rightarrow \alpha_w = 1\text{cm}^3\text{gm}^{-1} = 10^{-3}\text{m}^3\text{kg}^{-1}$ = constant

1 gm cm^{-3} = 1000 kg m^{-3}

Quantities and units

Quantity	**Derived unit MKS**	**MKS**	**Derived unit CGS**	**CGS**
Specific Vol	α	$\text{m}^3\cdot\text{kg}^{-1}$		
Force	Newton [N]	$\text{kg}\cdot\text{m}\cdot\text{sec}^{-2}$		
Force			Dyne [Dyn]	$\text{gm}\cdot\text{cm}\cdot\text{sec}^{-2}$
Pressure	Pascal [Pa]	$\text{N}\cdot\text{m}^{-2}$ OR $\text{kg}\cdot\text{m}^{-1}\cdot\text{sec}^{-2}$		
Pressure			Barye (ba)	Dyne cm^{-2}
Energy	Joule [J]	$\text{N}\cdot\text{m}$ OR $\text{kg}\cdot\text{m}^2\cdot\text{sec}^{-2}$		
			Erg	$\text{Dyne}\cdot\text{cm}$
Power	Watt [W]	$\text{J}\cdot\text{sec}^{-1}$ OR $\text{kg}\cdot\text{m}^2\cdot\text{sec}^{-3}$		

The ratio between a CGS unit and the corresponding MKS unit is usually a power of 10. A newton accelerates a mass 1000 times greater than a dyne does, and it does so at a rate 100 times greater, so there are 100 000 = 10^5 dynes in a newton.

peta (P) 10^{15}, tera (T) 10^{12}, giga (G) 10^9, mega (M) 10^6, kilo (k) 10^3, hecto (h)10^2, deca (da) 10^1, deci (d) 10^{-1}, centi (c) 10^{-2}, milli (mm) 10^{-3}, micro (m)10^{-6}, nano (n) 10^{-9}, pico (p) 10^{-12}, femto (f) 10^{-15}