CLOUD PHYSICS

Kelvin Equation: $r^* = \frac{2\sigma}{\rho_L R_v T \ln S}$ or $S = \exp\left[\frac{2\sigma}{\rho_L R_v T r}\right]$ | Where:

σ = surface tension, $\left[\frac{\text{force}}{\text{length}}\right] \Rightarrow \left[\frac{\text{N}}{\text{m}}\right]$, typical value: $0.075\ \frac{\text{N}}{\text{m}}$

ρ_L = density of H_2O, $\left[\text{kg m}^{-3}\right]$, value at sfc: 10^3 kg m^{-3}

S = supersaturation, expressed as ratio or %. S occurs when $\frac{e}{e_s} > 1$

$S_{\%} = (S_{ratio})\cdot 100 - 100$; $S_{ratio} = 1.01 = S_{\%} = 1\% =$ RH=101%

Equilibrium vapor pressure over a solution

Raoult's Law: gives the reduction in equilibrium vapor pressure due to dissolved salts etc. For dilute solutions, $n_s \ll n_w$ and $\frac{e'}{e_s(\infty)} = 1 - \frac{n_s}{n_w}$

$n_s = \frac{iN_0 m_s}{M_s}$; $n_w = \frac{N_0 m_w}{M_w}$

n_s = # of molecules of solute; n_w = # of molecules of water

e' = equilibrium vapor pressure over solution;

e_m = equilibrium vapor pressure over water

i = vant Hoff factor (~2)

N_0 = Avogadro's # = 6.022×10^{23} molecules mol^{-1}

m_s = mass of the solute; M_s = molecular weight of the solute

m_w = mass of water $= 4/3\pi r^3 \rho_L$; $M_w = 18$ g mol^{-1}

Raoult's Law for a spherical droplet

$$\frac{e_s'(r)}{e_s(\infty)} = 1 + \frac{a}{r} - \frac{b}{r^3}$$

AEROSOLS

Aitken, $r < 0.1\mu m$, $N_{land} = 10^3 - 10^6$ cm^{-3}, $N_{H20} = 10^2 - 10^3$

Large, $0.1 < r < 1$; $N_{land} = 10^2$; N_{H20} (funct of wind speed)

Giant, $r > 1\mu m$; $N_{land} = 1$; N_{H20} (funct of wind speed)

Coagulation: Simple, monodisperse

$$\frac{dN}{dt} = -KN^2 \Rightarrow \int_{N_0}^{N}\frac{dN}{N^2} = -\int_0^t K dt \Rightarrow N(t) = \frac{N_0}{1+N_0 Kt} \Rightarrow$$

$$K = \left(\frac{(N_0/N)-1}{tN_0}\right) \qquad t_f = -\frac{1}{k}\left(-\frac{1}{N} + \frac{1}{N_0}\right)$$

$N(t)\left[\text{cm}^{-3}\right]$ = Concentration after certain period of time

N_0 = cocentration at $t = 0$; t = time

$K = 8\pi rD$ = coag coefficient $\left[\text{Vol}/\text{t}\right]$ eg. $\left[\text{cm}^3/\text{sec}\right]$

$D = kTB = \frac{kT}{6\pi\eta r}$ = Diffusion coef of particle. Where k = Bolztman

$\therefore D$ is inversely $\propto$ to r.

$d(t) = d_0(1+N_0 kt)^{1/3}$ | d_0 = diameter at $t = 0$

Settling Velocity and Mechanical Mobility

$V_{TS}\left[\frac{\text{m}}{\text{s}}\right] = \frac{2}{9}\frac{\rho r^2 g}{\eta}$, | If $2r > 1\ \mu$m and $R_e < 1.0$; $\eta\left[\text{kg m}^{-1}\ \text{s}^{-1}\right]$ = dynamic viscosity, $\eta = \eta(\text{T})$, eg; $\eta(0°\ \text{C}) = 1.717\times10^{-5}$

Drag Force

$F_D[\text{N}] = 6\pi\eta vr$ | v = velocity, r = radius of particle, η = dynamic viscosity

Mobility = Velocity per unit force

$B\left[\text{m s}^{-1}\ \text{N}^{-1}\right] = \frac{v}{F_D} = \frac{1}{6\pi\eta r}$ or $V_{TS} = BF_D$ | If $2r > 1\mu$m

Specific Gravity $[\text{unitless}] = \frac{\rho_p}{\rho_w}$, | ρ_p = density of particle; ρ_w = density of liquid

Stokes number

$stk[\text{unitless}] = \frac{2}{18}\frac{\rho_L r^2 v}{R\eta}$ | where, r = radius of droplet, v = velocity, R = radius of collector, ρ_L = density of liquid; $\eta\left[\text{kg m}^{-1}\ \text{s}^{-1}\right]$ = dynamic viscosity

The smaller the radius of the collector, the higher the stk, & thus the higher the collection efficiency.

Stk $\approx \frac{r^2}{R}$

Reynolds number

$\text{Re} = \frac{\text{inertial force}}{\text{frictional force}} = \frac{2rV}{\upsilon}$, where ν = kinetmatic viscosity

Reynolds #: Re, after dimentional analysis => $R_e = \frac{2\text{rV}}{\nu}$

Liquid Water Content

$M\left[\text{kg m}^{-3}\right] = \rho_L \frac{4}{3}\pi r^3 N(r)$ | From Homework; $N(r)\left[m^{-3}\right]$ = droplet concentration

$M\left[\text{kg m}^{-3}\right] = \int \rho_L \frac{4}{3}\pi r^3 N(r)\, dr$ | From Prof. Goodman's notes

$$\Rightarrow \frac{dm(R)}{dt} = \frac{4}{3}\rho_L \pi 3r^2 \frac{dR}{dt} \Rightarrow \pi R^2 E V_R M = 4\rho_L \pi r^2 \frac{dR}{dt} \Rightarrow$$

$$\frac{dR}{dt} = \frac{\bar{E}M}{4\rho_L} \bullet V_T$$

Continuous Growth Equation

$\frac{dm(R)}{dt} = \pi R^2 E V_R M$, given m, calc: $dm = \_\_dr$

$\frac{dR}{dz} = -\frac{\bar{E}M}{4\rho_L}$ | For zero or negligibly small updraft; E = collection eff, $M\left[\text{kg m}^{-3}\right]$=liquid water content

$\frac{dR}{dt} = \frac{\bar{E}M}{4\rho_L} \bullet V_T$ | V_T = Terminal velocity

$\frac{dR}{dz} = \frac{\bar{E}M}{4\rho_L}\frac{V_T}{u - V_T}$ | For updraft. u = updraft; At top of cloud $\Rightarrow u = V_T$

$r < 30\mu m\,(3\times10^{-5}\ \text{m}) \Rightarrow V_T = k_1 r^2$, where $k_1 \approx 1.19\times10^6$ cm^{-1} s^{-1}

$40\mu m < r < 0.6$ mm $\Rightarrow V_T = k_3 r$, $k_3 = 8\times10^3$ s^{-1}

0.6mm $< r < 2$ mm $\Rightarrow V_T = k_2 r^{1/2}$, where $k_2 = 2.01\times10^3$ cm$^{1/2}$ s^{-1}

$k_2 = 2.01\times10^2$ m$^{1/2}$ s^{-1}

Note: 1 m$^{1/2}$ = 10 cm$^{1/2}$

Optics

$n=\frac{c}{v}$ (vel in vacuum/vel in medium). $n \approx 1/\lambda$; $n=n(T,\rho,\text{RH})$;

Terrestrial Refraction $1/R$ = curvature of the light ray

$\frac{1}{R}=79\times10^{-6}\frac{p}{T_v^2}(34-\gamma)$ | $R[\text{km}]$ = radius of ray curvature; p = pres $[\text{mb}]$; $\gamma[\text{C/km}]=-dT/dz$; T_v = virt T in K

$\frac{2\theta}{\Phi}=\frac{R_E}{R} \Rightarrow \theta=\frac{\Phi R_E}{2R}$ | $\Phi=\angle$ of arc length bordered by R_E; $\theta=\angle$ of refraction of light ray

or: $\theta=\frac{0.25\Phi p}{T_v^2}(34-\gamma)$ | where, $\gamma=-dT/dz$ C/km; T_v = virt T in K; p = pressure in mb

$\Phi[\text{rad}]=\frac{s}{R_E}$ | s = arc length

VISIBILITY

Steradian $[\text{sr}]=A/r^2$, sphere $\rightarrow 4\pi r^2/r^2 \Rightarrow 4\pi$ sr

1) Luminous Flux

Lumen $[\text{lm}]$ = 1 candle power/steradian; $[1 \text{ lm}=1.61\times10^{-3} \text{ W/sr}]$

(at $\lambda=0.56$ μm the power from a candle to which the eye is sensitive = 1 lm
1 W/sr = 621 lm

2) Illumination : Luminous flux density = energy flux/receiving area
$[1 \text{ phot}=1 \text{ lm/cm}^2]$. [1 lux = 1 meter-candle]: lux = luminous flux produced by 1 candle power at a distance of 1 m

3) Brightness = luminous flux / area of luminous sfc
$[1 \text{ Lambert}=1 \text{ lm/cm}^2]$ also see units of [meter lm]

BRIGHTNESS CONTRAST

$C\equiv\left|\frac{B_T-B_O}{B_O}\right|$ | B_T = Brightness of target/obj, B_O = Brghtness of backgrnd; B_r = apparent brightness of a black object

$C=-1\Rightarrow B_T=0$ (= black) in front of any background
$C=+\infty\Rightarrow$ non-black target in front of an absolutely black background
$C\in(0,-1)\Rightarrow$ dark target in front of light background
$C\in(0,+\infty)\Rightarrow$ light target in front of black background

$\varepsilon=\exp(-k_s y)$ = threshold contrast. $\varepsilon=0.02$ =TC for meteorology.

$Y=\frac{1}{k_s}\ln\left(\frac{1}{\varepsilon}\right)\Rightarrow Y=\frac{3.912}{k_s}$ | Y = Visual range; here for black target; In dry air, max distance $\approx$ 350 km

$B_r=B_h\left(1-e^{-k_s r}\right)$ Brightness for black target

$B_r=B_O e^{-k_s r}+B_h\left(1-e^{-k_s r}\right)$ | If non-black target, then its B must be added; B_O = brightness of object @ dist = 0

White target $\Rightarrow B_O\approx\frac{1}{2}B_h\Rightarrow -\frac{B_r-B_h}{B_h}=\frac{1}{2}e^{-k_s r}\Rightarrow Y=\frac{1}{k_s}\ln\left(\frac{1}{2\varepsilon}\right)$

Non-white target $\Rightarrow B_O\approx\frac{R}{2}B_h$ | R = reflectivity | Note: $\uparrow A\Rightarrow Y\downarrow$

Nighttime visual Range

$Y=\frac{1}{k_e}\left(\ln\frac{I}{E_m}-2\ln Y\right)$ | Allard's law. Y = dist at which E becomes E_m; E_m = threshold illuminance of the eye

$E_1=\frac{I_1 e^{-k_e x_1}}{x_1^2}$ | $E[\text{lm/m}^2]$ = Illumination, from a pnt source of Intensity, I at distance, x; k_e = extinction (abs + scatt) coefficient

$E_0=\frac{I_0}{x_0^2}$ $k=\frac{1}{x_1}\ln\left(\frac{I_1}{I_0}\right)\left(\frac{x_0}{x_1}\right)^2$

Acoustics : $f\lambda=v$ | $f[\text{Hz}]$ or $[\text{cyc/sec}]$ = frequency, λ=wavelength; v = velocity of sound wave

Source moving towards observer : $(f'\uparrow)$

$f'=\frac{v}{v-U}f$ | U = vel. of the source: $U<0\Rightarrow$ moving to obsrvr; $U>0\Rightarrow$ moving away. f' = freq as heard by obsvr

f = freq of sound as it emerges from the source.

$f'=\frac{v}{v+U}f$ | **Src moving away from obsrvr** $(f'\downarrow)$

$f'=\frac{v+U}{v}f$ | **Obsrvr moving to stat source** $(f'\uparrow)$

$f'=\frac{v-U}{v}f$ | **Obsrvr moving away from src** $(f'\downarrow)$

Sound vel. in dry air : $v=20.08\sqrt{T}$, $T[\text{K}]$ | $\therefore$ actual v = dry+hum

Humidity Correction : $dv_e=2.81\sqrt{T}\frac{e_s}{p}RH$ | $dv_e[\text{m/s}]$, $T[\text{K}]$, e_s = vapor pressure; p = pressure, $RH=e/e_s$ =humidity

Sound path in calm Atm : $\frac{\sin i_0}{v_0}=\frac{\cos\varepsilon_0}{v_0}=\frac{\sin i_1}{v_1}=\frac{\sin i_n}{v_n}=\text{const}$ | $i+\varepsilon=90°$

i = incidence$\angle$, ε = emerg$\angle$

Velocity at Apex : $V_a=\frac{v_0}{\sin i_0}$, **Temp at Apex :** $T_a=\frac{T_0}{\sin^2 i_0}$ | **Height of Ray,** $H=R(1-\sin i)$ ex: $H=9\text{K}(dT/dz)$

Radius of curvature as func of T & dT/dz | $R=\frac{2T_0}{\sin i_0(dT/dz)}$ | Std lapse rate $\Rightarrow R<0$; Denom = 0 $\Rightarrow R\rightarrow\infty$ | $-dT/dz=\gamma$

γ = 34C/km $\Rightarrow$ autoconvective; γ = 6.5C/km $\Rightarrow$ std lapse rate

$$R=\frac{v+3w\sin i}{\frac{10}{\sqrt{T}}\frac{dT}{dz}\left(\sin i-\frac{w}{v}\cos(2i)\right)+\frac{dw}{dz}\left(1+\frac{w}{v}\sin^3 i\right)}$$

v = sound vel in calm air corrsponding to T

- $\cap$ toward ground can occur when T & $w\uparrow$ w/ z if denom $>0\Rightarrow R>0$
- $\cap$ toward grnd can also occur w/ std γ if denom's RHS $>-$LHS $\Rightarrow R>0$
- $\cap$ occurs if $w_2>w_1>w_0$; if $w_2<w_1<w_0\Rightarrow\cup$ {consider effect of w only}

$R_v=\frac{v}{\frac{dw}{dz}-\frac{w}{2T}\frac{dT}{dz}}$ | Radius of curvature for vertical ray. $R_v=\infty$(straight); if $\frac{1}{w}\frac{dw}{dz}=\frac{1}{2T}\frac{dT}{dz}$. | $\therefore$ effect of ∇T on sound prop. is 2X greater than that of ∇w

Attenuation of sound : absorption coefficient, $k[\text{cm}^{-1}]$:

$k[\text{cm}^{-1}]=1.6\times10^{-16}f^2/\rho$ | f = frequency, ρ = density of air. v = sound vel.

$k=1.4\times10^6 Lf^2/v^3$ | L = mean free path of molecules, $\sim10^{-5}$cm @ SeaLevel

Molecular attenuation:

$I=I_0e^{-kx}\Rightarrow -k=\frac{2.303}{x}\log_{10}\frac{I}{I_0}$, $\alpha=\log_{10}\frac{I}{I_0}$ [bel]; $\alpha=10\log_{10}\frac{I}{I_0}$[decibel]

Threshold level: $I_0=10^{-10}\text{ W/m}^2$; Threshold of audibility of ear:
$\alpha=1, I=10I_0$; $\alpha=2, I=100I_0$;

Propagation of sound in the Stratosphere

$\frac{\sin i_0}{v_0}=\frac{\sin 90}{V_A}\Rightarrow V_A=\frac{v_0}{\sin i_0}$ | Vel @ Apex. now need to find i_0; How? by setting up 2 stations?

$\sin i_0=\frac{v_0\Delta t}{\Delta s}=20.08\sqrt{T}\frac{\Delta t}{\Delta s}\Rightarrow\sin i_0=\frac{v_0}{V_A}\Rightarrow V_A=\frac{v_0}{\sin i_0}\Rightarrow V_A=\frac{\Delta s}{\Delta t}$

OPTICS : $n = \frac{c}{v}$ (vel in vacuum/vel in medium). $n \approx 1/\lambda$; $n = n(T, \rho, \text{RH})$;

High $T \Leftrightarrow$ low ρ & low n, Stronger $\nabla T \Leftrightarrow$ Stronger ∇n & greater refraction

Linear $dt/dz \Rightarrow$ light takes parabolic path. Rays always bend so that the cold (denser air) is on inside of curve, $\therefore$ img is displaced in the direction of warm (less dense) air.

Curvature of a light ray as a function of lapse rate

$\frac{dT}{dz}\uparrow$	>11C/100m	concave downward	Ray curvature > Earth's
	11C/100m	concave downward	Ray curvature = Earth's
	<11C/100m	concave downward	Ray curvature < Earth's
	0	concave downward	Ray curvature < Earth's
$\frac{dT}{dz}\downarrow$	<3.4 C/100m	concave downward	Ray curvature < Earth's
	3.42 C/100m	straight line (autoconvective)	
	>3.4 C/100m	concave up	

Note: $\gamma = -dT/dz$

Normal lapse rate: γ=6.5C/km or $dT/dz = -6.5$C/km. $\Rightarrow$ Light rays are never straight since n depends on ρ & $\rho\downarrow$ w/ z. To get straight ray of light, need autoconvective γ, which only occurs near the surface on hot days.

If $dT/dz < -34.2$C/km, $\rho\uparrow$ w/ z, heavier air sits above lighter, rays are bent $\cup$

The curvature of the light ray, ie. the reciprocal of the radius, R, can be computed:

$$\frac{1}{R} = 79\times10^{-6}\frac{p}{T_v^2}(34-\gamma)$$ where, $\gamma = -dT/dz$ C/km; T_v = virt temp in Kelvin, p = pressure in mb

$T_v = T(1+0.61\cdot r)$, where r = water vapor mixing ratio (m_v/m_d)

For normal γ, $1/R = 2.96\times10^{-5}$ km

$1/R = 1/R_e$ when $T\uparrow$ w/z at rate of 110 C/km

Atm produces **Mirages** by refraction, by gradual variations of n

I) $+dt/dz$ (**T inversions**): • Superior img.; • Ray curvature: $\cap$

1) Looming: A'oB'=AoB; constant $dT/dz \Rightarrow$ no img magnification

2) (Rare) Towering: $(1/R)_A > (1/R)_B$, $\nabla T\uparrow$ as $T\uparrow$ (w/ $\uparrow z$) $\Rightarrow$ img magnified near sfc, $dt/dz < 0$ (pt B), then curve changes and aloft @ A, $dt/dz > 0$

3) Stooping: $(1/R)_A < (1/R)_B$, A'oB'<AoB; $\nabla T\downarrow$ as $T\uparrow$ (w/ $\uparrow z$) $\Rightarrow$ img reduction: ie. bottom of obj is seen thru stronger ∇ than top, will be lifted more than top.

II) $-dt/dz$: • Inferior img; • Ray curvature: $\cup$

4) Sinking: constant dt/dz $\downarrow$>34C/km $\Rightarrow$ no img magnification

5) (Majority of) Towering: $\nabla T\uparrow$ as $T\uparrow \Rightarrow$ img magnified, ie. max T & max ∇T found at bottom of T profile, both $\downarrow$ w/ $\uparrow z$; bottom of obj. will be displaced more than top b/c bottom is seen thru stronger ∇T.

2 - Image (inverted) Mirage : $\nabla T\uparrow$ as $T\uparrow$, tho T profile has a greater curvature, $\Rightarrow$ ray that travels through ∇T becomes so strngly bent produces 2nd inv. img. To give rise to 2 images instead of single towering one, T profile must have a somewhat greater curvature. In an inferior mirage, the efffect can be accomplished by an increase in the temp gradient at the surface of the ground or of the water. A ray of light that travels thru this region off strong Temp gradient becomes so strongly bent that it will no longer be able to join the eye with the botttom of some distant object but will instead join the eye w/ the top of th object to give a second, inverted image. The image is inverted b/c as the observer lifts his gaze slightly, they are looking thru a region of the atm that has a weaker temp gradient, so that the ray is less strongly curved. It will $\therefore$ join the eye to a point lower on the object rather than higher, as would usually bee expected.

Shimmering & lack of sharpness are due to small irregularitiees in density & temp that result from turbulence in the air.

REFRACTION : Halo is a ring of light encircling & extending out from sun/moon, produced when sun or moonlight is refracted as it passes through ice crystals, indicating the presence of cirriform clouds. The most common is the 22° halo, a ring of light 22°from the sun, formed when tiny column ice crystals (d < 20 μm) refract light. Blue: outside

Dispersion is the breaking up of white light by "selective" refraction: red (longer λ) slows the least & $\therefore$ bends the least; violet (shorter λ) slows the most & $\therefore$ bends the most. $\therefore$ as light travels through ice crystals, dispersion causes red light to be on the inside of the halo & blue on the outside.

Sun Dogs (parhelia) occur when sun, ice crystals, & obeserver are on ~ same horizontal plane, causing appearence of a pair of bright spots on either side of sun, red (bent least) on inside (sun side) & blue (bent more) on outside.

REFLECTION : Sun Pillars are caused by reflection of sunlight off of fallling ice crystals

Rainbows - light gets redirected back to observer: as light enters raindrop, it slows & bends, w/ red refracting the least & violet the most. The light ray is internally *reflected* when it strikes the backside of the drop at an angle > the critical angle for water (H_2O $\angle_{crit}$ = 48°). Refraction of the light as it enters the drop causes the point of reflection (on back side) to be different for each color, $\therefore$ colors are separated from eachother when light emerges (red $\angle_{emerg}$ = 42°, violet = 40°). When violet light from a lower drop reaches us, the red from that drop is incident at our waist,

Optics, Rainbows (con't)

$\therefore$ b/c red comes from higher drops, & violet from lower, the colors of primary bow change from/appear as red on outside (top) & violet on inside (bottom). 2ndary bow is caused when light enters drops at an $\angle$ that allows the light to make 2 internal reflections in each drop, causing the colors to reverse from primary. NOTE: only 1 ray of light is able to enter your eye from each drop. With each movement, light from different raindrops eneters your eyes.

Diffraction : Coronas occur when the moon is seen thru a thin veil of clouds made of tiny spherical H_2O droplets, due to *diffraction :* bending of light as it passes around objects. When light waves constructively interfere, we see bright light; destructive $\Rightarrow$ darkness. Colors appear when the cloud droplets (or aerosols) are of uniform size. B/c the amount of bending due to diffraction depends upon the λ, the shorter λ blue light appears on the inside of a ring, while the longer λ red appears on the outside. The smaller the cloud droplets, the larger the ring diameter. $\therefore$ clouds that have recently formed (such as thin altostratus & altocumulus) are the best corona producers.

Glory is also a diffraction phenomenon: appears as a bright ring of light around the shadow of person/object. Sun must be at one's back, so that sunlight can be returned to your eye from the water droplets. Sunlight that enters the droplet along its edge is refracted, then reflected of the backside of the drop. The light then exits at the other side of the droplet, being refracted once again. However, in order for the light to be returned to your eyees, the light actually clings to the edge of the droplet, it actually skims along the surface of the droplet as a surface wave for a short distance. Diffraction of light coming from tthe edges of the droplet produces the ring of light we see as the glory. The colorful rings are due to the various angles at which different colors leave the droplet.

Constants & useful Values :

Radius of Earth = 6.370×10^6 m

Total charge of Earth's sfc : -5×10^5 C; **Surface density of charge :** 10^{-9} C/m^2

Conductivity of air (at sea level): 2×10^{-16} (Ω cm)$^{-1}$

Current Density (j) : over sea: conductivity due to small ions; City: large ions

Conduction current = j: (Air to Earth) 2-4 $\times10^{-16}$ A/cm^2

Intensity of the electric field at the surface: 130 V/m

Total air - Earth current (whole globe): 1800 A

Potential Diff between Earth & Ionosphere : 3.6×10^5 V

Breakdown potential (dry air): 30000 V/cm. (in clouds) 10000 V/cm

$\downarrow$ w/ $\downarrow p$ & $\downarrow w$/ $\uparrow$ moisture in droplets

Potential diff between neg lightning leader & earth > $\times10^7$ V

Electric Field changes in lightning : • Currents in a return stroke rise to max 20,000 A in a few μs • Max: 260000 A, max duration 200 ms (hot lightning)

• Charge transfer by flash: ~ 25 C • Channel Temp: 10000 - 40000 K

• Core conductivity: ~ 2×10^{4} (Ω m)$^{-1}$

Quantity	**Representative value**	**Range**
Length of leader step	50 m	3-200 m
Time between steps	50 μsec	$30-125$ μsec
Leader propagation vel.	1.5×10^5 m/sec	$10^5 - 2.6\times10^6$ m/sec
Velocity of datr leader	2×10^6 m/sec	$10^6 - 2\times10^7$ m/sec
Vel of return stroke	5×10^7 m/sec	$2\times10^7 - 1.4\times10^8$ m/s
Channel length	5 km	$2-14$ km
Return strokes/path	3-4	$1-26$
Time between ret. strokes	40 m sec	3-100 m sec
Time duration of entire flash	0.2 sec	0.01- >2 sec
Charge transfer by flash	25 C	$1-200$C
Channel temp		$10^4 - 4\times10^4$ K

Math Essentials

$\frac{d}{dx}(e^{ax}) = ae^{ax}$ $\int e^{ax}dx = \frac{e^{ax}}{a}$ Law of cosines: $a^2 = b^2 + c^2 - 2bc\cdot\cos A$

Arc length: $s = r\theta$, (θ in radians), $1° = \frac{\pi}{180}$ rad; 1 rad $= \frac{180°}{\pi}$; $\text{Vol}_{\text{sphere}} = \frac{4}{3}\pi r^3$

Terrestrial Refraction $\frac{2\theta}{\Phi} = \frac{R_E}{R}$ $\theta = \frac{\Phi R_E}{2R}$

$$\theta = \frac{0.25\Phi p}{T_v^2}(34-\gamma)$$ where, $\gamma = -dT/dz$ C/km; T_v = virt T in Kelvin, p = pressure in mb

Electric Mobility $= k[\text{m}^2\ \text{s}^{-1}\ \text{V}^{-1}]$ $k = \frac{V_{TE}}{E} = \frac{NeC_e}{3\pi\eta d}$ N = # of elementary charges on particle, $e = q$ = charge, C_e = slip factor (~1.2)

η = dynamic viscosity (1.766×10^{-5} kg m^{-1} s^{-1}

Electrostatic Force $F_E = NeE$: $F_E[\text{C V m}^{-1}]$, $E\left[\frac{\text{V}}{\text{m}}\right] = \frac{\partial V}{\partial z} \approx \frac{\Delta V}{\Delta z}$ = Field Intensity

Terminal Electrostatic Velocity $u = kE$, u[m/s]

ELECTRICITY - Coulomb's Law - force between two point charges

$F = \dfrac{Q_0 Q}{\varepsilon r^2}$ where, Q & Q_0 are charges, r = distance betwen charges

ε = permittivity; $\varepsilon_0 = 10^7 \left(4\pi c_0^2\right)^{-1}$ C^2 N^{-1} m^{-2}; in air: $\varepsilon \approx \varepsilon_0$

$Q_0 Q = F\varepsilon r^2$; $Q[\text{C}]$ or $[\text{esu}]$; $F[\text{N}]$ or $[\text{dyne}]$

- 1 esu (electrostatic unit) of charge exerts 1 dyne of force on an equal charge 1 cm away in a vacuum. $\therefore$ esu = $\left[\text{dyne}^{1/2}\ \text{cm}\right]$ or $\left[\text{g}^{1/2}\ \text{cm}^{3/2}\ \text{s}^{-1}\right]$ Electrostatic problems involve charges at rest.

$e = 1.6\times10^{-19}$ C, magnitude of the charge of an electron or proton

4.803×10^{-10} ~~esu~~ $\left(\dfrac{3.336\times10^{-10}\ \text{C}}{1\ \text{esu}}\right) = 1.6\times10^{-19}$ C

Coulomb (C) - unit of electric charge; Ampere = 1 C/s; 1V = 1J/C

Electric field at a certain point is equal to the *electric force per unit charge* experienced by a charge at that point. Avg fair weather $\vec{E} \approx 130$ V/m;

$\vec{E} = \dfrac{\vec{F_0}}{q_0}$ $\vec{E}$[N/C] or [V/m] or $\left[\text{g}^{1/2}\ \text{cm}^{-1/2}\ \text{s}^{-1}\right]$ $\vec{F_0} = q_0\vec{E}$, force exerted on a point charge q_0 by an electric field $\vec{E}$

If q_0 is *positive*, the force $\vec{F_0}$ experienced by the charge is the same direction as $\vec{E}$; if q_0 is *negative*, $\vec{F_0}$ & $\vec{E}$ are in opposite directions.

Electric Field Intensity/ : $E_z = 130$ V/m (fair weather) $E_z = \dfrac{\partial V}{\partial z}$ In Atm, V gets more + w/ z

E_z[V/m] = (**vertical**) **Potential Gradient :** varies diurnally w/ dust & H_2O e, max (19 GMT) & min (3GMT) occur @ same t over globe, in sync w/ t-storms

Electric Potential [Volts] represents the energy level of the charges, ie. their ability to do work. Physically, the difference between the electric potential at any 2 points is the amount of work done by an external force when moving a unit charge at constant velocity along any path connecting these 2 points. EP-that function of position ϕ the negative gradient of which is the (static) electric field $\vec{E} = -\nabla\phi$; Potential gradient $\downarrow$ exp. w/ z

Potential difference, V = potential energy per unit charge.

PD=$\int E_z = \int \dfrac{\partial V}{\partial z} \Rightarrow$ $V = \int E_z dz$ or $\Delta V = E_z \Delta z$ or $\Delta V = \dfrac{j}{\lambda}\Delta z$

$V = \dfrac{Q_0}{\varepsilon r}$; $V\left[\text{g}^{1/2}\ \text{cm}^{1/2}\ \text{s}^{-1}\right]$ or $\left[\dfrac{\text{ergs}}{\text{esu}}\right]$; or $V\left[\dfrac{\text{kg m}^2}{\text{A s}^3}\right]$; $V = \dfrac{\text{work}}{\text{unit charge}}$

Volt [W/A] or [J/C] is the SI derived unit of electric potential difference or voltage. **Voltage** is a difference in potential from one point to another

Current, i, is the rate of flow of a charge at a point, in the direction in which there is a *positive* charge. $i = \dfrac{dQ}{dt} = neuA$ Q = total # of ions * e; $Q = ne$ * volume of box = $neAut$

$I = \left[\dfrac{\text{C}}{\text{s}}\right]$ or [Amp] or $\left[\dfrac{\text{esu}}{\text{s}}\right]\left[\dfrac{\text{g}^{1/2}\text{cm}^{3/2}\text{s}^{-1}}{\text{s}}\right]$ $n\left[\text{m}^{-3}\right]$ = concentr; $I = i$; A = X-sectional area; $e = q$

$n = \dfrac{Au}{\text{vol}_{\text{drop}}}$, $n\left[\dfrac{\#}{\text{time}}\right]$; $Q = qn$; Intensity of rain = u[m/s]; $\text{Vol}_{\text{box}} = \text{Area}\cdot u\cdot t$ $\text{mass} = \rho\cdot\text{vol}_{\text{drop}}$

$\dfrac{\#\text{ of electrons}}{\text{drop}} = \text{charge/drop} \big/ \text{charge of 1 electron}$

Current density, J = current per unit cross-sectional area.

$J = \dfrac{I}{A} = neu$; $J\left[\dfrac{\text{A}}{\text{m}^2}\right]$ u = drift velocity; $e = q$ = charge $J = J_+ + J_-$

Since ions can be either + or −, there will be 2 streams of ions in an electric field, each moving in opposite directions & each contributing to J.

Resistivity $\rho = \dfrac{E}{J}$; $\rho\left[\dfrac{\text{V m}}{\text{A}}\right] = [\Omega\ \text{m}]$ or [s] where 1 Ω (ohm) = 1V/A; $\rho = \kappa$ (Met125)

The greater the resistivity, the greater the field needed to cause a given current density, or the smaller the current density caused by a given field.

$\rho = 0 \Rightarrow$ perfect conductor; $\rho = \infty \Rightarrow$ perfect insulator

Conductivity depends mainly on small ion n, since their mobility $\gg$ large ion's

Conductivity is the ; reciprocal of resistivity $\lambda = 1/\kappa$ $\left[(\Omega\ \text{m})^{-1}\right]$ or $\left[\text{A V}^{-1}\ \text{m}^{-1}\right]$ or $\left[\text{s}^{-1}\right]$; $\lambda = nek$ $\left[(\Omega\ \text{m})^{-1}\right]$ or [esu]

Variation over land: min @ 8am & 8pm; max before sunrise.

PBL: λ varies inversely w/ aerosol or large ion N. $\lambda\uparrow$ w/ z, varies inv $\vec{E}$; Seasonaly(urban) : greater in summer than winter. Ocean: small variation.

Above PBL, λexponential $\uparrow$ due to cosmic rad, $\downarrow$ in aerosol, $\uparrow$ mobility w/ z

Resistance $R = \dfrac{V}{I} = \dfrac{\rho L}{A}$ $R[\Omega] = \left[\dfrac{\text{V}}{\text{A}}\right]$ where L = length, A = Area

Specific Resistance : $A\dfrac{\partial R}{\partial z}$, **Specific Conductivity :** $\lambda = \dfrac{i}{dV/dz}$, $\lambda = \dfrac{1}{A}\dfrac{\partial z}{\partial R}$ reciprocal of spec res.

Ohm's Law $V = IR$ OR $E_z = j_z \dfrac{1}{\lambda}$, $j_z = \lambda E_z$ 2 alternate forms of Ohm's law, better for Atm. formed from doing substitutions in: $E_z = \dfrac{\partial V}{\partial z} = i\dfrac{\partial R}{\partial z} = \dfrac{i}{A}\left(A\dfrac{\partial R}{\partial z}\right)$ j_z = current; j_z = **conduction current**

Ionic Mobility = the terminal velocity of the ion in a unit electric field. The mobility of intermediate ions is 2 orders of magnitude smaller than that of small ions, & the diff is much greater for large ions, $\therefore$ small ions carry almost all of the current. Mobility = avg drift v under potential gradient of 1V/m $\Rightarrow \left[\text{m s}^{-1}/\text{V m}^{-1}\right] \Rightarrow \left[\text{m}^2/\text{V s}\right]$; conductivity depends on n of small ions

Max Ion production at 12 km due to a balance of air density & how many high energy particles can penetrate. Cosmic rays (Or cosmic radiation): rays of ET origin that continually bombard the earth and consist mostly of high-energy protons, about 9% helium and heavier nuclei, a small percentage of electrons, and some gamma rays.

Ion Concentrations Small ions: e^+ ~480/cm^3; e^- ~425/cm^3 - clean air land & ocean N negative ions slightly < than $N+$ ions b/c many negative are free electrons and they have higher mobility than positive and they become attached to the other charged or neutral particles so that they have a shorter lifespan. This is why the atmosphere is positively charged. N of large ions of opposite charge are almost equal. $n = p\tau$, p = rate, τ=time

Vertical distribution: Total ion concentration increases from ground to about 6 km then decrease. Small ions decrease after 1km then increase in height and reach a max at 12 km~tropopause. This distribution is due to ion production and destruction (sfc: radioactive decay, in upper atm: cosmic rays from outer space.)

Global Electric Circuit : B/c vertical current density (which = $(\vec{E})$ · elec. conductivity) must be same @ all levels, $\vec{E}$ must $\uparrow$ if conductivity $\downarrow$. @ z > 100 m, $\vec{E}\downarrow$ & conductivity $\uparrow$ b/c of $\uparrow$ in ionization by cosmic rays w/ z, as well $\downarrow$ N of large particles. The presence of the downward directed fair weather $\vec{E} \Rightarrow$ that electrosphere carries net + charge & Earth's sfc net $-q$

Global Electrical budget $\left[\text{C km}^{-2}\ \text{year}^{-1}\right]$

- 90 units of + charge gained from fair weather conductivity
- 30 + units gained from precip; • 100 + units lost thru pt. discharges
- 20 + units lost due to transfer of neg charges to the Earth by ground lightng

Lightning Types: ⊙ Sheet L is the area illumination w/out appearance of stroke ⊙Heat L is lighning beyond the audibility of thunder. ⊙ Bead L appears as a chain of luminous points, usually as a residual of an ordinary lighning stroke ⊙ Ball L is a sphere from few inches to few feet, some pulsate ionized plasma ⊙ More than 1/2 of all flashes are intercloud (IC) ⊙ **Cloud ↔ Ground**: 4 types, 90% are neg Cloud to Ground (CG), sequence:

- t = 1 ms, prelim breakdown in cloud leads to stepped leader (SL). occurs when vel. of free electrons becomes sufficient to ionize by collision, $\therefore$ forming an electron avalanche. • t = 1.1, SL propagates $\downarrow$ in a series of discrete steps (L: tens of m., duration :1 μs, pause time: 20-50 μs) • SL lowers tens of C of neg charge to Earth in tens of ms.
- $\bar{v}$ of propagation is 2 $\times10^5$ m/s. • Avg leader current is 100 – 1000 A, w/ peak pulse currents of at least 1 kA., producing typical downward-branched structure • t = 20 ms, attachment process w/ ground. • t = 20.1 ms, first return stroke • t = 40 ms, K & J processes • t = 60 ms, first dart leader goes down ionized channel; dart leader has peak current of 1 kA or more, & lowers 1 C at a speed of 3×10^6 m/s • t = 62.05 ms, 2nd return stroke

Notes: positive leaders are faster than neg. During lighnting events, conduction currents above the cloud & corona currents below it transfer charge from the env. to the cloud at rates of Coulombs/sec. After one cloud becomes elcetrified, others nearby can as well. **Elements of the current paradigm :**

Charging mechanism : 1) Cloud is elec. neutral, then charge separation takes place 2) Charge separation occurs from collisions between: • large & small ice particles, where charge of one sign is transferred to large, & equal & opposite to small • falling precip & smaller cloud particles. 3) when faster-falling larger precip particles fall away from small cloud particles, large-scale charge sep results w/ exensive $\vec{E}$ that bring about dielectric breakdown & lightning. 4) The electrical energy responsible for lightning is derived from the fall of charged precip particles under the influence of gravity. 5) The −charge residing on the surface of the earth in fair weather is the result of many thunderstorms continuosly in progress. 6) Fair-weather electrical phenomenon do not have sig. infuence on thunderstorm electrification. 8) Effect of lighting is to neutralize the charged particles resp. for electrifcation. Notes: Obs show that significant electrification rarely occurs before the appearanceof precip.

Phase changes: • During freezing, ice becomes neg w/ respect to water. • During melting, charge sep occurs. • Condensation: + charge; evap: neg

Temp differentials : charging bewteen 2 ice surfaces when there is a difference in temp or N of dissocated contaminants. **Mechanical shearing or stress :** Electrification produced by the rupture of large drops (in strong updraft): large fragments carry + charge. Opposite to thunderstorm. **Collisions between cloud & precip particles :** • drop is + charge on botttom, neg on top. falling drop collects neg ions on outer bottom edge, making whole drop net neg charge.

- Electrification associated collision & fracture of ice crystals: fracture is +

CONVERSION FACTORS

Temperature :

$\left(\frac{9}{5}\times{}^\circ\mathrm{C}\right)+32={}^\circ\mathrm{F}$; $\left({}^\circ\mathrm{F}-32\right)\times\frac{5}{9}={}^\circ\mathrm{C}$; $\mathrm{K}={}^\circ\mathrm{C}+273.15$

Area :

$1\ \mathrm{cm}^2=10^{-4}\ \mathrm{m}^2 \Leftrightarrow 1\ \mathrm{m}^2=10^4\ \mathrm{cm}^2$

Volume : $\mathrm{V}=1$ liter $=10^3\mathrm{cm}^3=10^{-3}\mathrm{m}^3$

$1\ \mathrm{m}^3=10^6\mathrm{cm}^3 \Leftrightarrow 1\mathrm{cm}^3=10^{-6}\ \mathrm{m}^3$

Force :

$1\ \mathrm{Dyn}=10^{-5}\ \mathrm{N} \Leftrightarrow 1\ \mathrm{N}=10^5\ \mathrm{Dyn}$

Energy :

1 calorie $=4.18684\times10^7$ erg $=4.18684$ Joule

1 Joule $=10^7$ erg $\Leftrightarrow$ 1 erg $=10^{-7}$ J

$1\ \mathrm{J\ gm^{-1}} = 1000\ \mathrm{J\ kg^{-1}}$

Pressure :

1atm = 1013.25mb = 1013.25hPa = 101.325kPa = 1.01325×10^5Pa = 1.01325 bar

1 hPa = 100 Pa; 1 mb $=10^2$ Pa

Density : $\rho=\frac{m}{V}$; $\alpha=\frac{V}{m}$; $\Rightarrow V=\alpha m$; $\therefore\ \rho=\frac{m}{\alpha m}=\frac{1}{\alpha}$; $\Rightarrow\alpha=\frac{1}{\rho}$

$\rho_w=1\mathrm{gm}\cdot\mathrm{cm}^{-3}$; $\Rightarrow \alpha_w=1\mathrm{cm}^3\mathrm{gm}^{-1}=10^{-3}\mathrm{m}^3\mathrm{kg}^{-1}$ = constant

$1\ \mathrm{gm\ cm^{-3}}=1000\ \mathrm{kg\ m^{-3}}$

Quantities and units

Quantity	Derived unit MKS	MKS	Derived unit CGS	CGS
Specific Vol	α	$\mathrm{m}^3\cdot\mathrm{kg}^{-1}$		
Force	Newton [N]	$\mathrm{kg}\cdot\mathrm{m}\cdot\mathrm{sec}^{-2}$		
Force			Dyne [Dyn]	$\mathrm{gm}\cdot\mathrm{cm}\cdot\mathrm{sec}^{-2}$
Pressure	Pascal [Pa]	$\mathrm{N}\cdot\mathrm{m}^{-2}$ OR $\mathrm{kg}\cdot\mathrm{m}^{-1}\cdot\mathrm{sec}^{-2}$		
Pressure			Barye (ba)	Dyne cm^{-2}
Energy	Joule [J]	$\mathrm{N}\cdot\mathrm{m}$ OR $\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{sec}^{-2}$		
			Erg	$\mathrm{Dyne}\cdot\mathrm{cm}$
Power	Watt [W]	$\mathrm{J}\cdot\mathrm{sec}^{-1}$ OR $\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{sec}^{-3}$		

The ratio between a CGS unit and the corresponding MKS unit is usually a power of 10. A newton accelerates a mass 1000 times greater than a dyne does, and it does so at a rate 100 times greater, so there are 100 000 = 10^5 dynes in a newton.

Numerical values

N_A is Avogadro's number = 6.022×10^{23} molecules mol^{-1} = 6.022×10^{26} molecules kmol^{-1}

1 gram-mole of any gas contains 6.0220943×10^{23} molecules; 1 mole = .001 kilomole

$M_v=18.016$ amu

$R^*=8314.3\ \mathrm{J}\cdot\mathrm{kmole}^{-1}\cdot\mathrm{K}^{-1}$ | Universal gas constant

$R^*=8.314.3\ \mathrm{J}\cdot\mathrm{mole}^{-1}\cdot\mathrm{K}^{-1}$

$R_d=287.05\ \mathrm{J}\cdot\mathrm{kg}^{-1}\cdot\mathrm{K}^{-1}$; $R_v=461.5\ \mathrm{J}\cdot\mathrm{kg}^{-1}\cdot\mathrm{K}^{-1}=4615\mathrm{mb}\cdot\mathrm{cm}^3\cdot\mathrm{gm}^{-1}$(check)

Reference values (dry air)

$p_0=1.01325\times10^5$ Pa

$\rho_0=1.225\ \mathrm{kg}\cdot\mathrm{m}^{-3}$

$c_p=1004\ \mathrm{J\ kg^{-1}\ K^{-1}}=1.00464\ \mathrm{J\ gm^{-1}\ K^{-1}}$ (const for IG)

$c_v=717\ \mathrm{J\ kg^{-1}\ K^{-1}}=0.7176\ \mathrm{J\ gm^{-1}\ K^{-1}}$, $c_v=\left(\frac{\partial u}{\partial T}\right)_\alpha$ (for any substance)

$\kappa=2/7=0.286$ (for dry air or IG)

$\gamma=c_p/c_v=7/5=1.4$, value for IG or dry air.

w & μ are usually $\le 0.04\,\mathrm{gm/gm}$ (= 40 gm/kg = 40 parts/1000)

$\eta=\eta(T)$; $\eta(0)=1.766\times10^{-5}\ \mathrm{kg\ m^{-1}\ s^{-1}}$

- Condensation $\Rightarrow$ latent heat release $\Rightarrow$ parcel temp $\uparrow$ $\Rightarrow$ saturation mixing ratio $\uparrow$
- ΔT due to condensation: $\Delta T=\frac{l(\Delta w)}{c_{pd}}$ | w = mixing ratio